Bloch-Zener oscillations to probe Dirac points merging in artificial graphene

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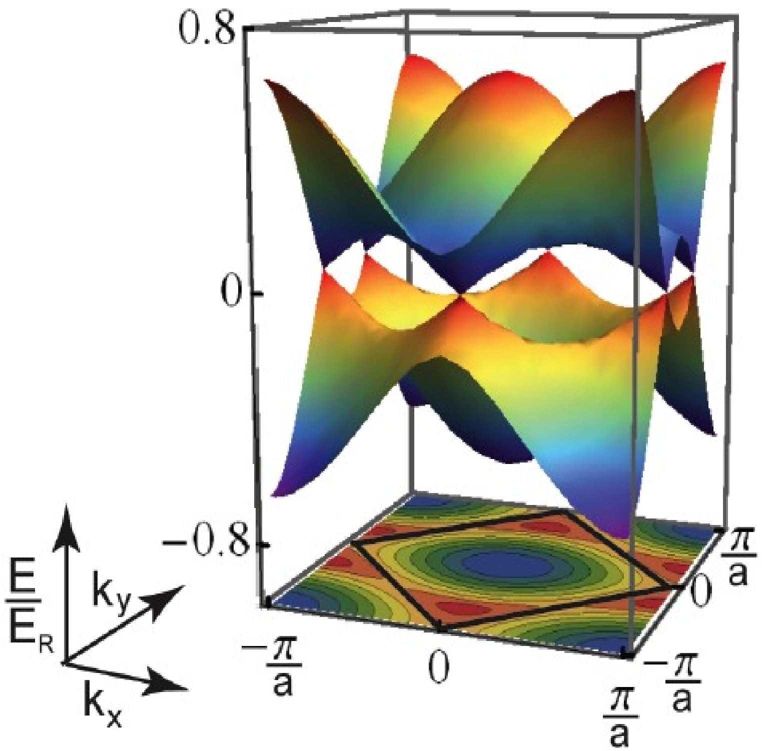

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Outline

1. Dirac points in 2D crystals: graphene and others

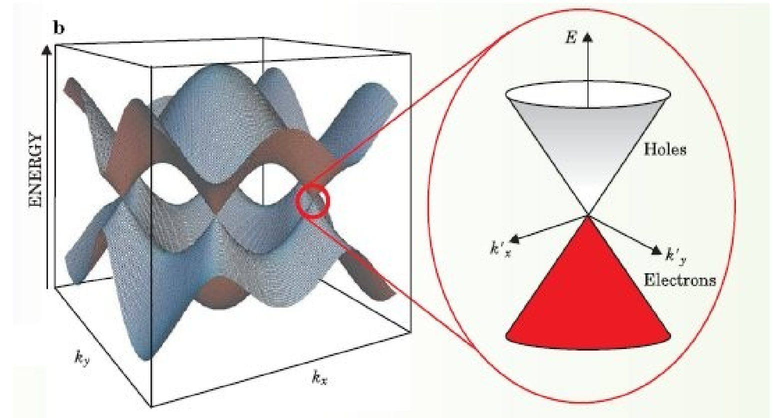
2. Motion and merging of Dirac points

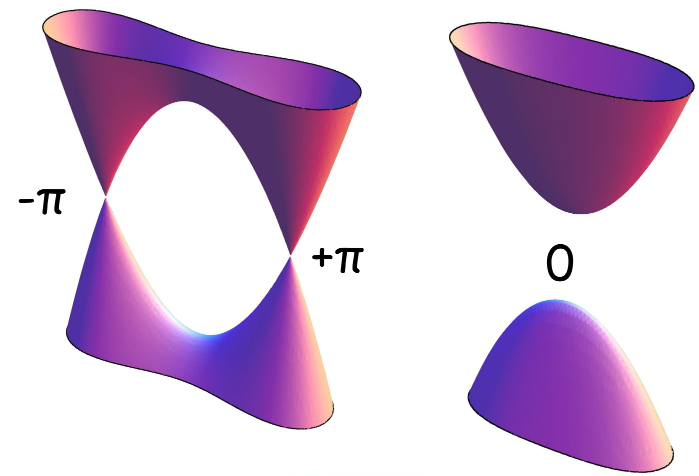
3. Physical realizations of the merging

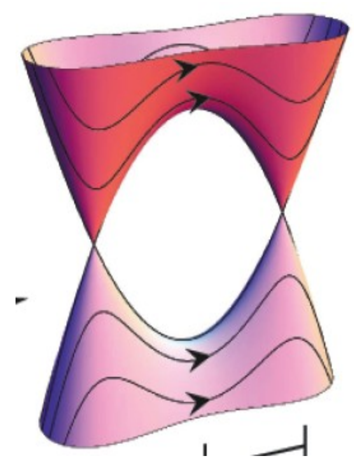
4. Bloch oscillations and Landau-Zener tunneling as a probe of Dirac points:
the ETH Zürich experiment

5. Bloch-Zener oscillations across a merging transition:
the Orsay theory

6. Perspectives

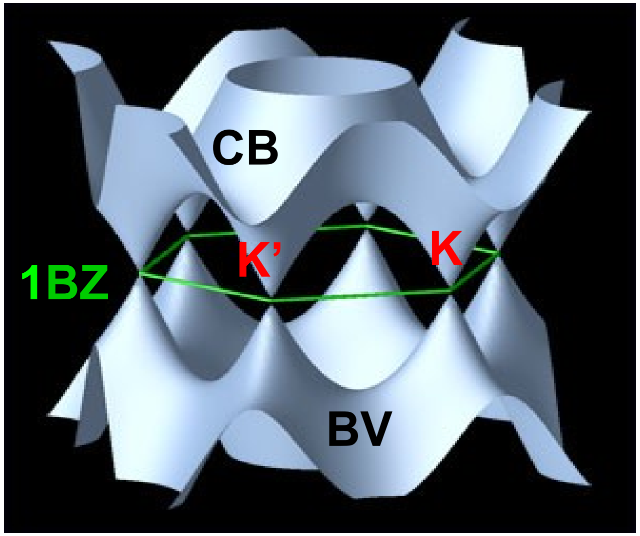






1. Dirac points in 2D

Dirac points = linear band crossing points
Description by a 2D massless Dirac (Weyl) hamiltonian
Carries a topological charge ±1: a winding number for the pseudo-spin ½ (or quantized Berry phase ± π)
Usually in pairs (fermion doubling)

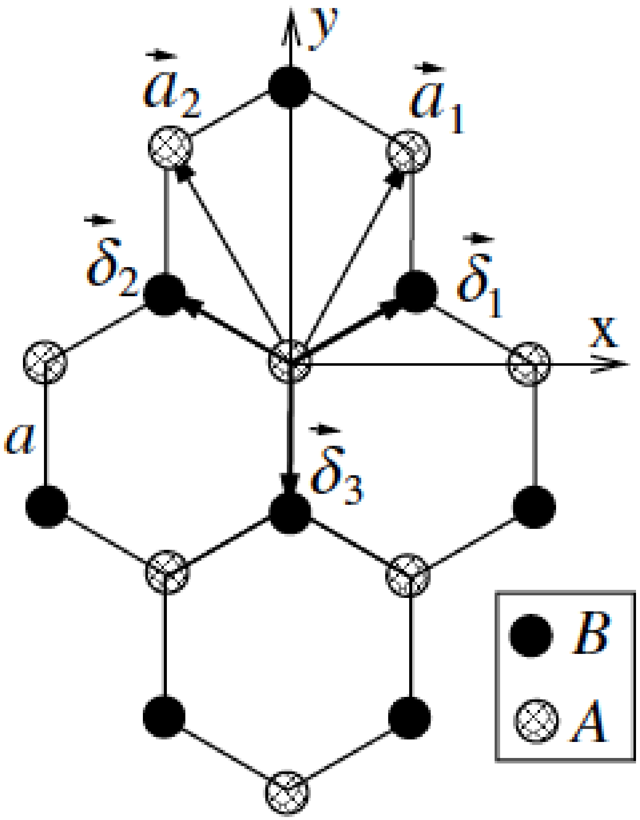


Examples:
Graphene: honeycomb lattice
Other 2D lattices (brick-wall, kagome, dice,
Nodal points in d-wave superconductors
Surface states in 3D topological insulators

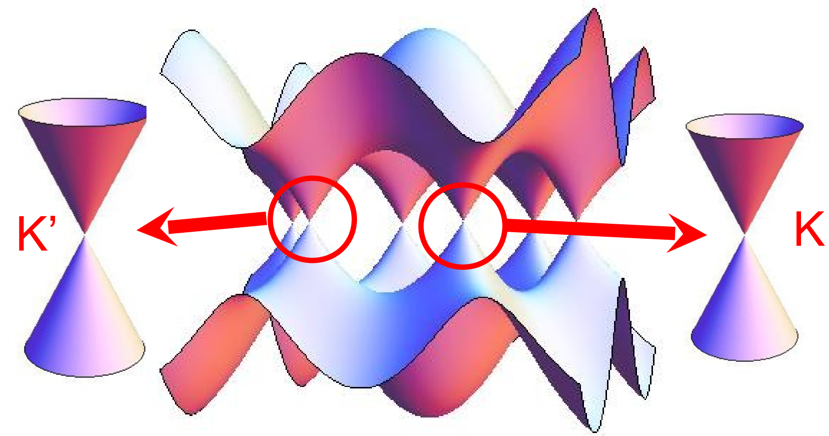
Stability:
Inversion or time-reversal symetry breaking opens a gap.
Otherwise topological stability similar to that of an elementary vortex (here in k-space). Removal of Dirac points is via annihilation or merging of a +1/-1 pair. A weak perturbation does not open a gap (critical threshold).

Honeycomb lattice: tight-binding and Dirac hamiltonians

$$f(\mathbf{k}) \equiv -t(1 + e^{-i\mathbf{k}\cdot\mathbf{a}_1} + e^{-i\mathbf{k}\cdot\mathbf{a}_2}) = |f(\mathbf{k})|e^{-i\theta_\mathbf{k}}$$



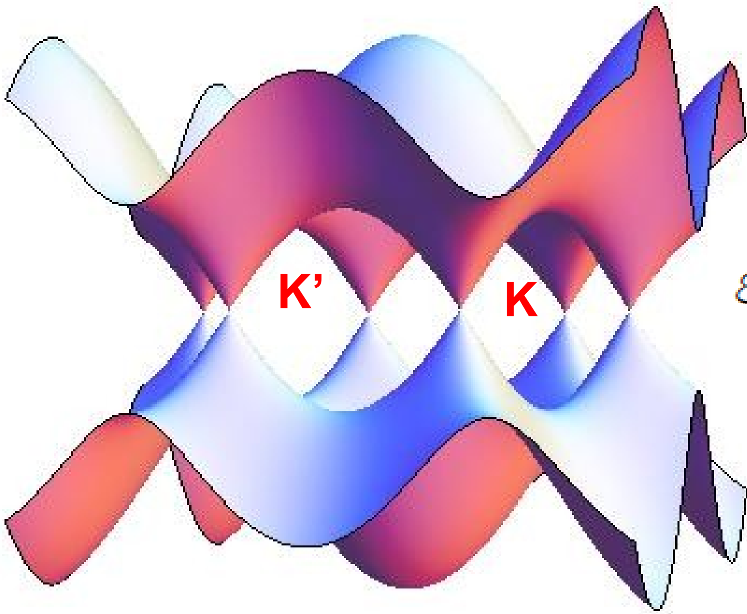
$$H(\mathbf{k}) \equiv \begin{pmatrix} 0 & f(k) \\ f(k)^* & 0 \end{pmatrix}$$



$$\begin{aligned} f(\mathbf{k}_\xi + \mathbf{q}) &= f(\mathbf{k}_\xi) + \mathbf{q}\cdot\nabla_\mathbf{k} f(\mathbf{k}_\xi) + \ldots \\ &= \xi\frac{3at}{2}q_x - i\frac{3at}{2}q_y + \ldots \end{aligned}$$

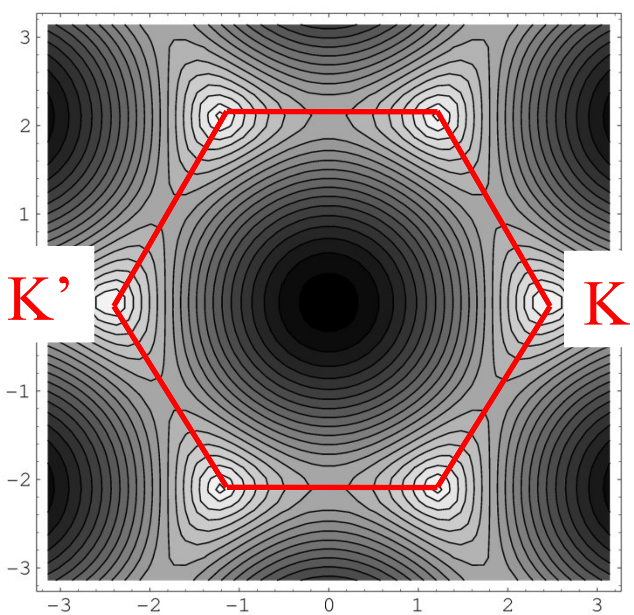
$$H_\xi(\mathbf{k}) = \hbar v_F(\xi q_x\sigma_x + q_y\sigma_y) = \xi\hbar v_F q \begin{pmatrix} 0 & e^{-i\xi\theta_\mathbf{q}} \\ e^{i\xi\theta_\mathbf{q}} & 0 \end{pmatrix}$$

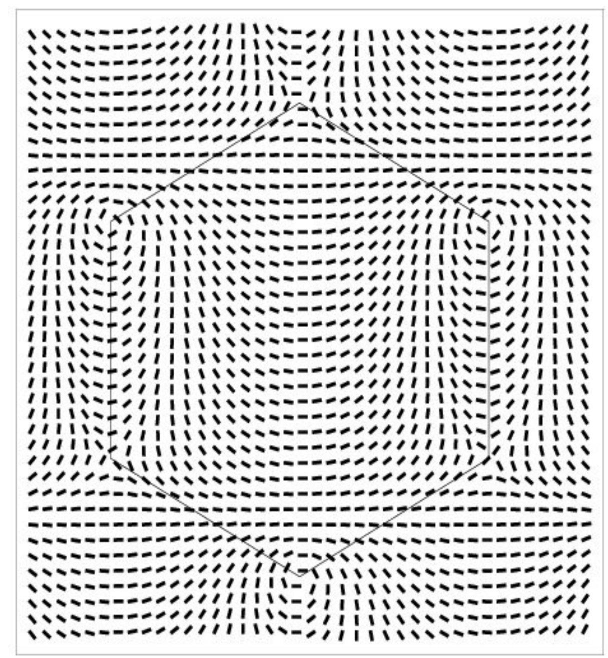
There is more to the hamiltonian than its spectrum



$$\varepsilon_{\mathbf{k}} = \pm|f(\mathbf{k})|$$

$\theta_{\mathbf{k}}$





$+\pi$

$-\pi$

2. Manipulation of DP and merging

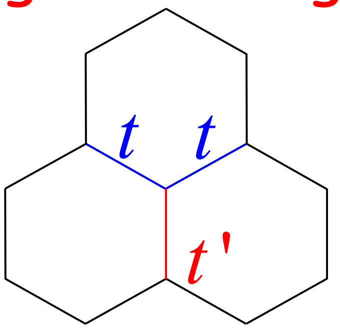
It is possible to manipulate the Dirac points. They can move in k-space and they can even merge.

Motion through varying band parameters such as hopping amplitudes (physically: lattice deformation via strain, electron interactions, laser intensity in optical lattices, etc.)

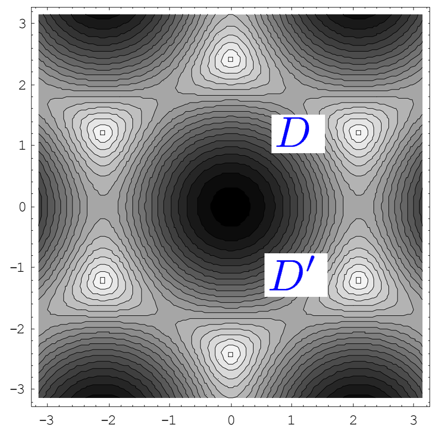
The merging transition is a topological Lifshitz transition: 2 Dirac points become a single « hybrid » band crossing point and eventually a gap opens and the Fermi surface disappears.

Hasegawa, Konno, Nakano, Kohmoto, PRB 2006
Dietl, Piéchon, Montambaux, PRL 2008
Wunsch, Guinea, Sols, NJP 2008
Montambaux, Piéchon, Fuchs, Goerbig, PRB 2009 and EPJB 2009
See also A. Kitaev, Ann. Phys. 2006 and Volovik, Lect. Notes in Phys 2007

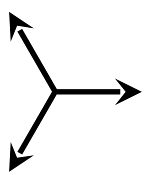
Tight-binding problem on anisotropic honeycomb lattice

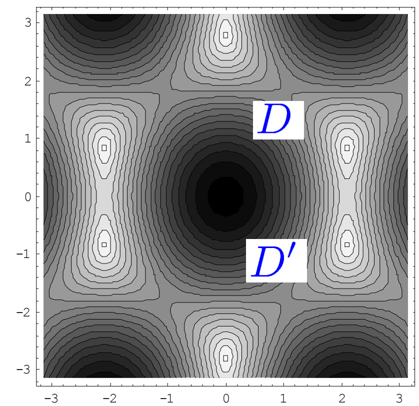


$$E = |t' + t\,\exp(ika_1) + t\,\exp(ika_2)|$$

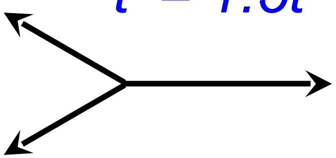


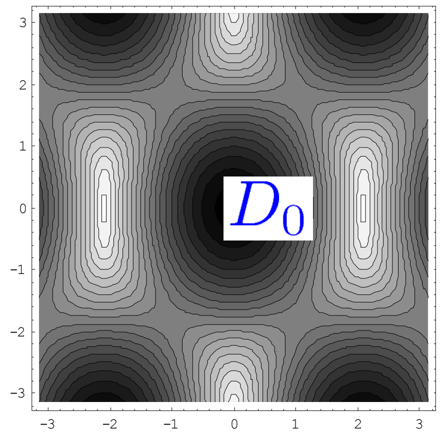
$t' = t$





$t' = 1.5t$

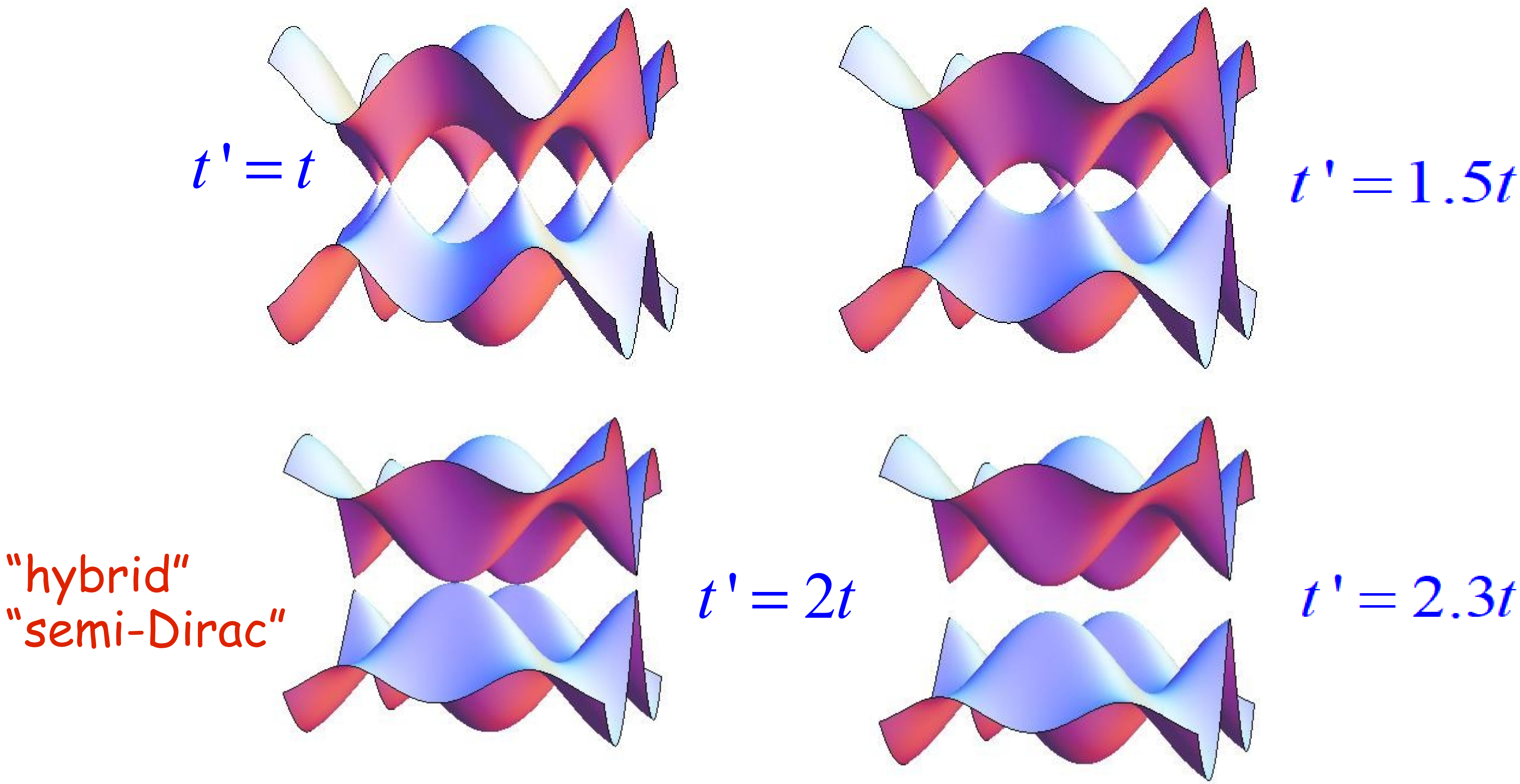




$t' = 2t$

Y. Hasegawa et al., PRB 2006

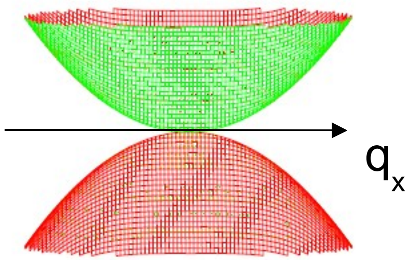
Motion and merging of Dirac points



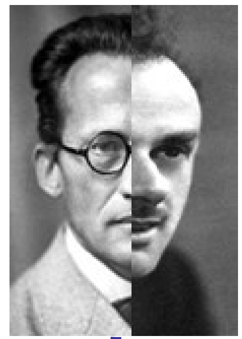
Hybrid contact point : a peculiar dispersion relation

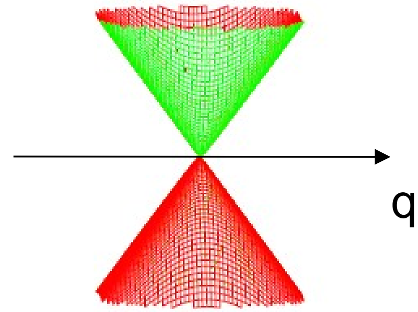
$$t' = 2t$$

Schrödinger

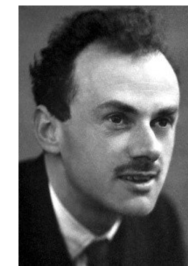


Semi-Dirac





Dirac



$$\varepsilon = \pm\sqrt{\frac{q_x^4}{4m^2} + c^2 q_y^2}$$

Dietl, Piéchon, Montambaux, PRL 2008
Pardo, Pickett, PRL 2009

General description of the vicinity of the merging transition

2D crystal with 2 sites per unit cell (A and B), any Bravais lattice
Time-reversal and inversion symmetry

$$\mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} 0 & f(\boldsymbol{k}) \\ f^*(\boldsymbol{k}) & 0 \end{pmatrix} \qquad f(\boldsymbol{k}) = \sum_{m,n} t_{mn} e^{-i\boldsymbol{k}\cdot\boldsymbol{R}_{mn}}$$

$$\boldsymbol{R}_{mn} = m\boldsymbol{a}_1 + n\boldsymbol{a}_2$$

When the hopping amplitudes t_{mn} change, the Dirac points $\boldsymbol{D}$ and $-\boldsymbol{D}$ move

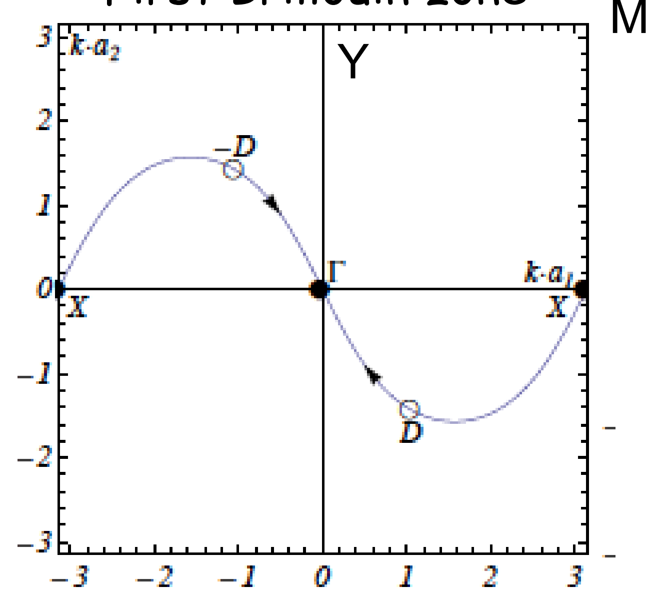
Where is the merging point?

$$\boldsymbol{D} = -\boldsymbol{D} \longrightarrow \boldsymbol{D}_0 = \boldsymbol{G}/2$$

4 possible positions in $\boldsymbol{k}$ space

(0,0)	(0,1)	(1,0)	(1,1)
Γ	X	Y	M

First Brillouin zone



Expansion near $\boldsymbol{D}_0$ gives

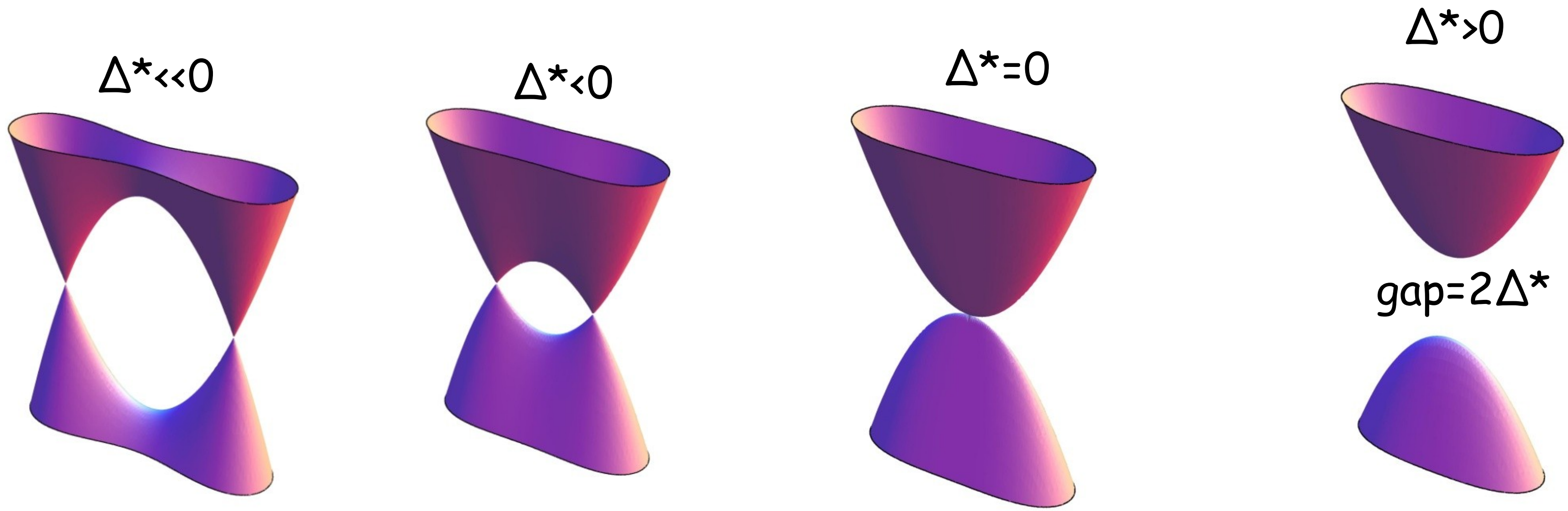
$$f(\boldsymbol{D}_0 + \boldsymbol{q}) = \frac{q_x^2}{2m^*} - ic_y q_y$$

Montambaux, Piéchon, Fuchs, Goerbig, PRB 2009 and EPJB 2009

« universal Hamiltonian »

$$\mathcal{H}(\boldsymbol{q}) = \begin{pmatrix} 0 & \Delta_* + \frac{q_x^2}{2m^*} - ic_y q_y \\ \Delta_* + \frac{q_x^2}{2m^*} + ic_y q_y & 0 \end{pmatrix}$$

The parameter Δ* drives the topological transition



This Hamiltonian describes the topological transition, the coupling between valleys and the merging of the Dirac points

Montambaux, Piéchon, Fuchs, Goerbig, PRB 2009 and EPJB 2009

3. Physical realizations of the merging transition

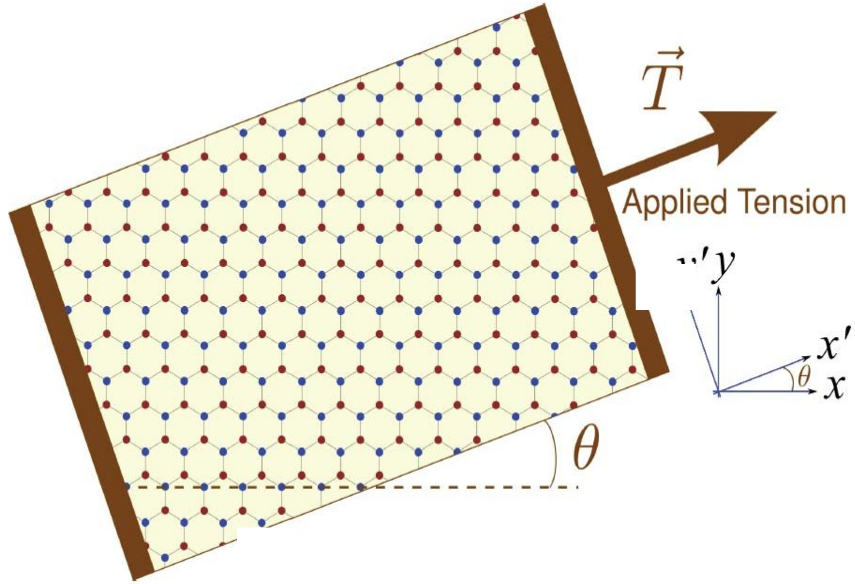
Strained graphene

Quasi-2D organic salts α-$(BEDT\text{-}TTF)_2I_3$ under pressure Katayama, Kobayashi, Suzumura, JPSJ 2005

Artificial « graphene »:
- with ultracold atoms in optical lattices
- with semiconductors superlattices Gilbertini et al., PRB 2009

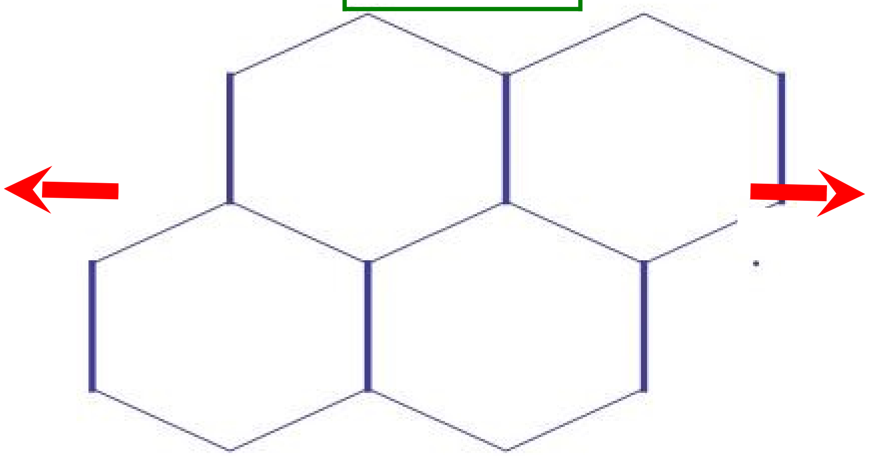
TiO_2/VO_2 nanostructures Pardo, Pickett, PRL 2009

Strained graphene: merging is unreachable

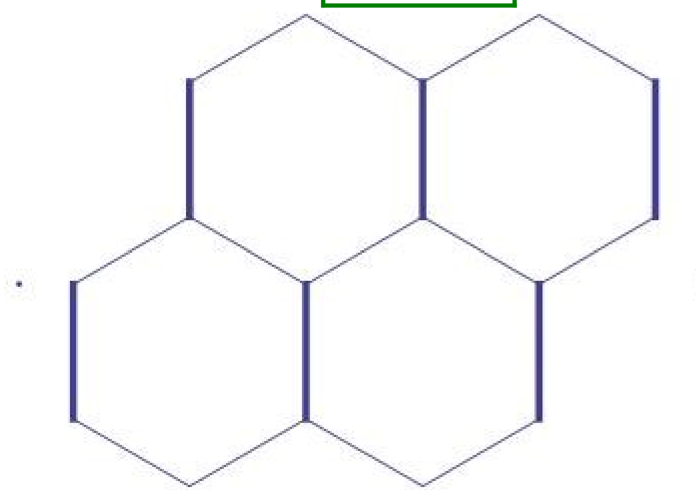


Merging transition:
strain ~ 23%

$t' = 2t$



$t' = t$



Pereira, Castro Neto, Peres, PRB 2009
See also Goerbig, Fuchs, Piéchon, Montambaux, PRB 2008

« Graphene » with ultracold atoms in optical lattices

PRL **98**, 260402 (2007) PHYSICAL REVIEW LETTERS week ending 29 JUNE 2007

Simulation and Detection of Dirac Fermions with Cold Atoms in an Optical Lattice

Shi-Liang Zhu,[1,3] Baigeng Wang,[2] and L.-M. Duan[3]

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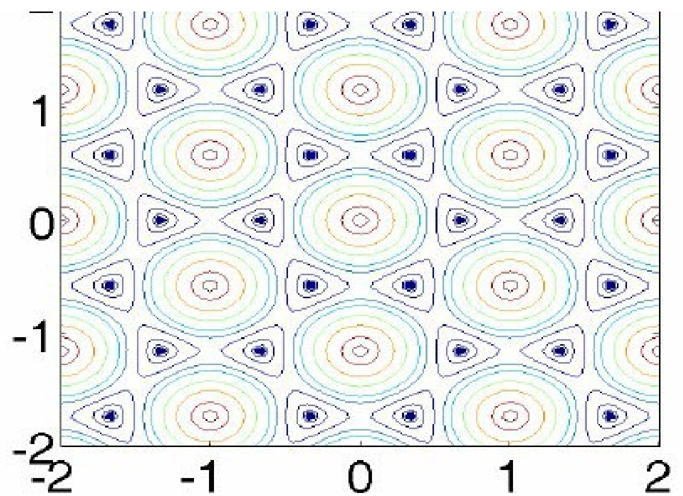
[2]*National Laboratory of Solid State Microstructures and Department of Physics, Nanjing University, Nanjing, China*
[3]*FOCUS Center and MCTP, Department of Physics, University of Michigan, Ann Arbor, Michigan 48109, USA*

(Received 24 March 2007; published 25 June 2007)

We propose an experimental scheme to simulate and observe relativistic Dirac fermions with cold atoms in a hexagonal optical lattice. By controlling the lattice anisotropy, one can realize both massive and massless Dirac fermions and observe the phase transition between them. Through explicit calculations, we show that both the Bragg spectroscopy and the atomic density profile in a trap can be used to demonstrate the Dirac fermions and the associated phase transition.



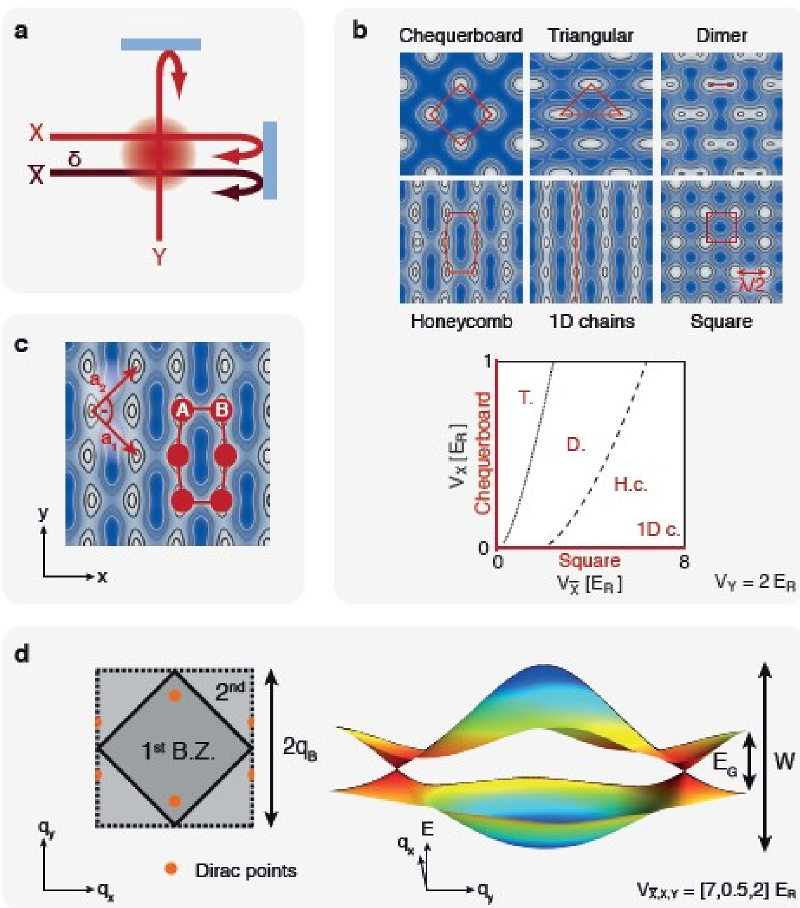
Theory:
Zhao, Paramekanti, PRL 2006
Zhu, Wang, Duan, PRL 2007
Wunsch, Guinea, Sols, NJP 2008
Lee, Grémaud, Han, Englert, Miniatura, PRA 2009
Bahat-Treidel, Peleg, Grobman, Shapira, Pereg-Barnea, Segev, PRL 2009
Hou, Yang, Liu, PRA 2009

4. Bloch oscillations and Landau-Zener tunneling as probes of Dirac points: ETH experiment

Creating, moving and merging Dirac points with a Fermi gas in a tunable honeycomb lattice

Leticia Tarruell, Daniel Greif, Thomas Uehlinger, Gregor Jotzu and Tilman Esslinger
Institute for Quantum Electronics, ETH Zurich, 8093 Zurich, Switzerland

Nature 2012



Ultracold atoms
3D degenerate Fermi gas (^{40}K) T ~ 0.2 E_F

Harmonic trap
2D optical lattice → tubes

$$V(x,y) = -V_{\overline{X}} \cos^2(kx + \theta/2) - V_X \cos^2(kx)$$
$$-V_Y \cos^2(ky) - 2\alpha\sqrt{V_X V_Y} \cos(kx)\cos(ky)\cos\varphi$$

Tunable optical lattice: V_X, V_{Xb}, V_y

Checkerboard
Triangular
Brick-wall (« honeycomb-like »
1D chains
Square

Applied constant force (electric field like)

Creating, moving and merging Dirac points with a Fermi gas in a tunable honeycomb lattice

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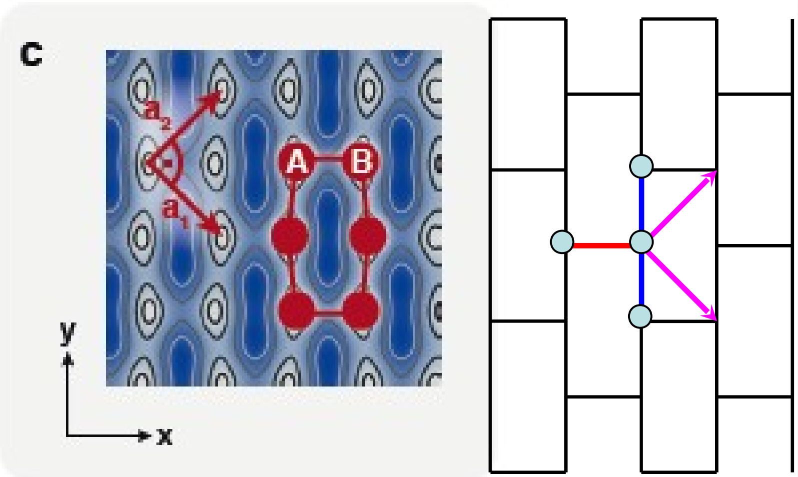
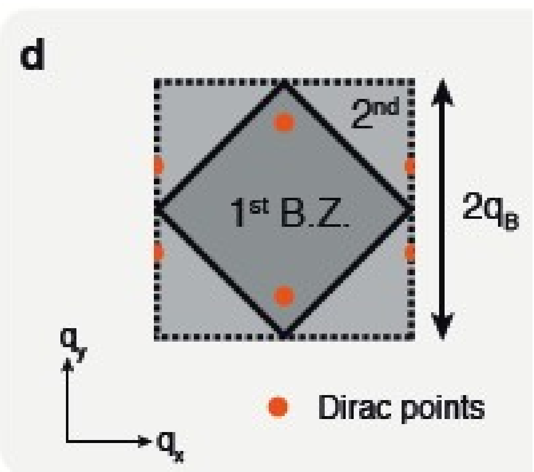
Institute for Quantum Electronics, ETH Zurich, 8093 Zurich, Switzerland

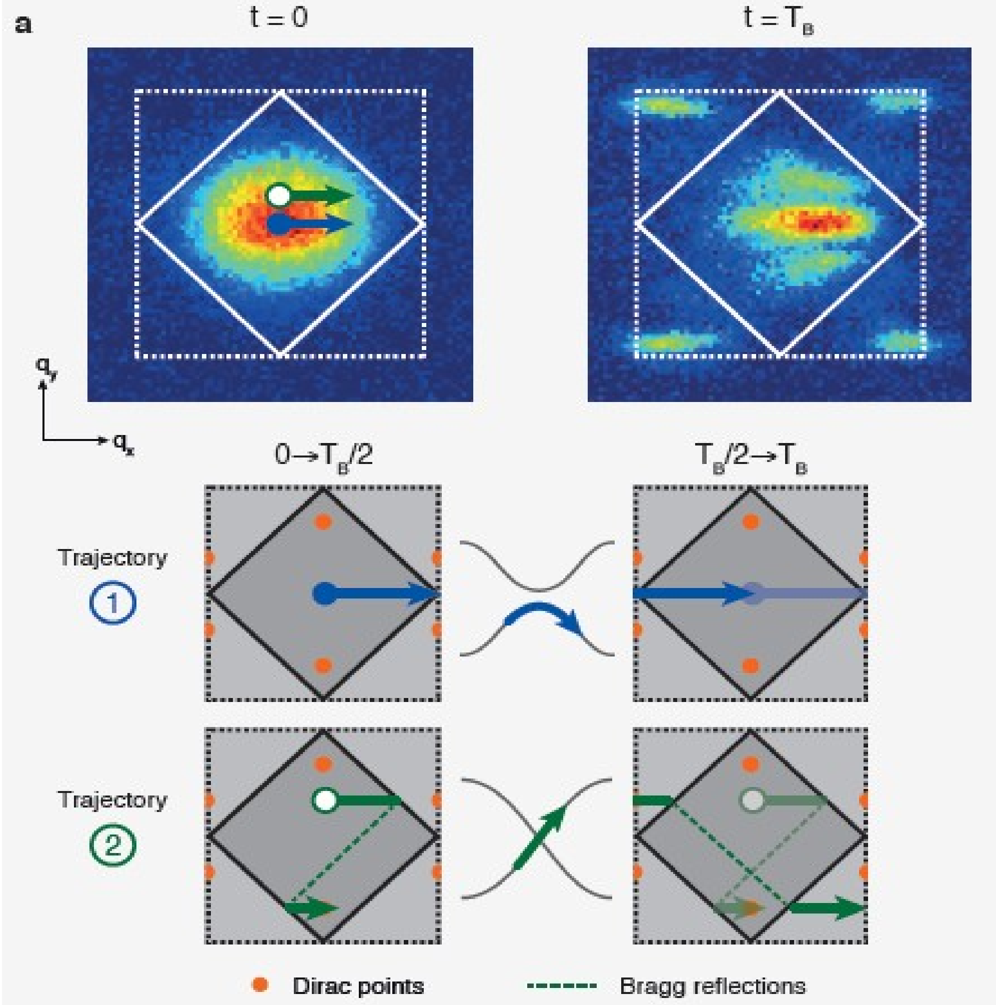
Nature 2012

Brick-wall: Dirac points

Bloch oscillations







How to manipulate and merge Dirac points ?

↓

Anisotropy of the optical potential

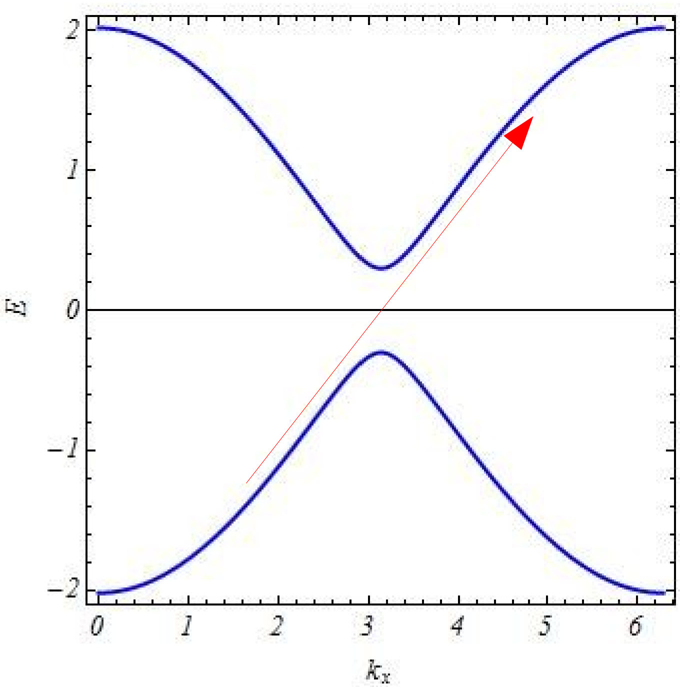
How to detect and localize Dirac points ?

↓

Bloch oscillations + Landau-Zener tunneling

Landau-Zener tunneling

Landau, Zener, Stückelberg, Majorana 1932



Jumping (non-adiabatic) probability

$$P_Z^x = e^{-\pi \frac{(\text{gap}/2)^2}{c_x F}}$$

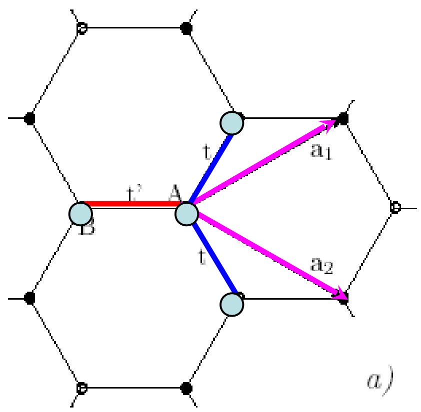
Staying (adiabatic) probability

$$1 - P_Z$$

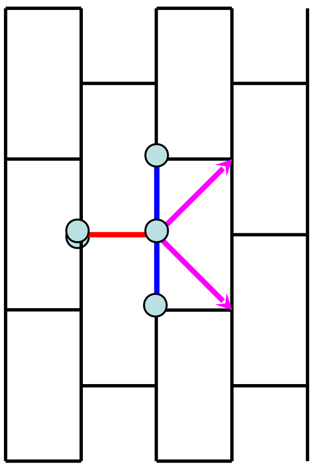
Applied force *F*
Velocity in the direction of motion c_x
Minimum gap to the upper band

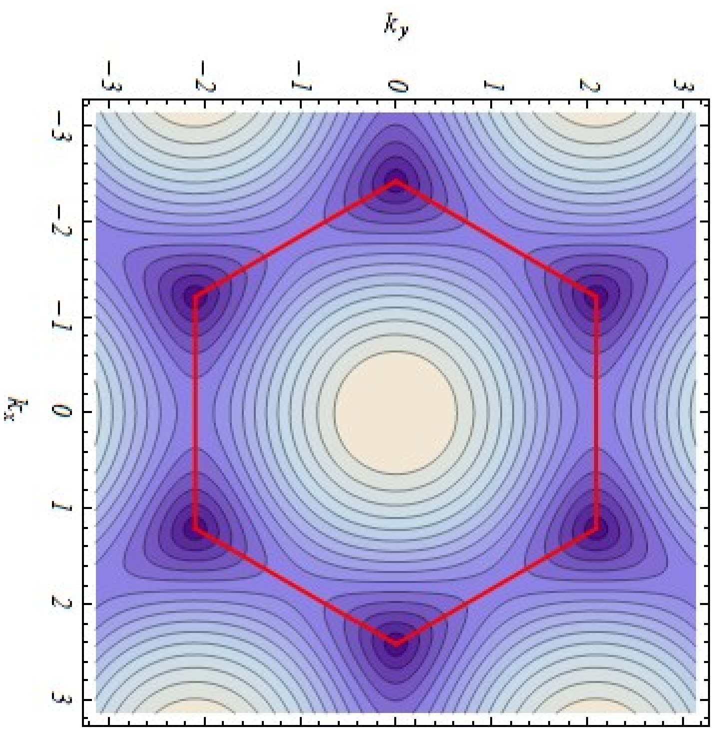
Remark: 100% interband transition probability (Landau-Zener) exactly at a Dirac point. Same as Klein-Sauter tunneling in graphene. Due to pseudo-spin conservation.
Review on Klein tunneling: Allain, Fuchs, EPJB 2011

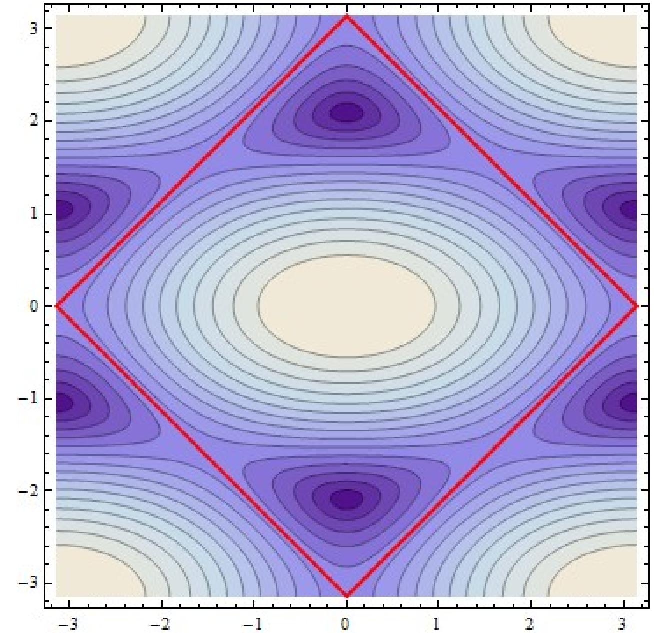
Honeycomb lattice



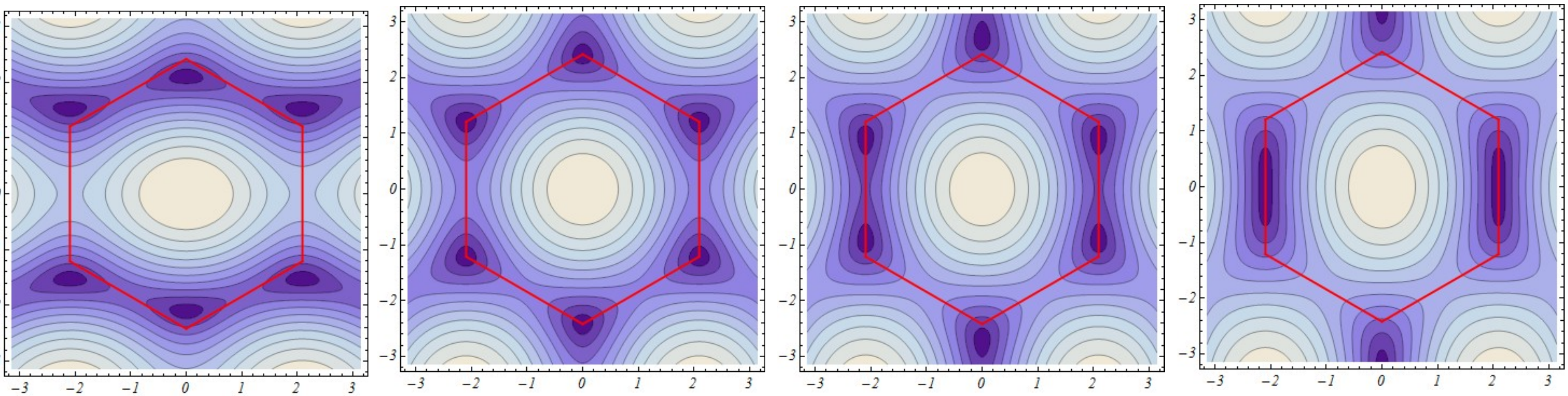
Brick-wall lattice





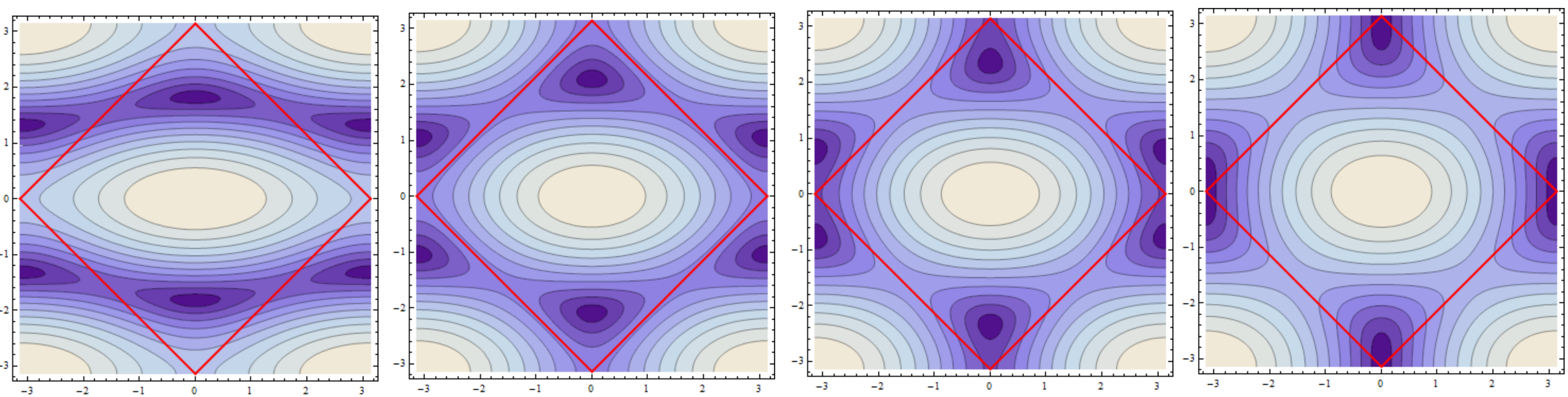


Honeycomb lattice



t=0.5 | t=1 | t'=1.414 | t'=2

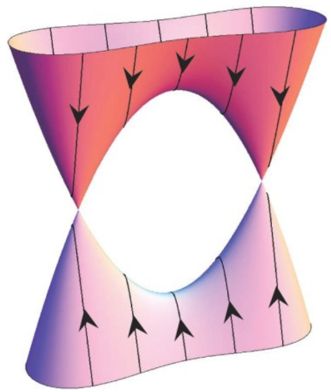
Brick-wall lattice

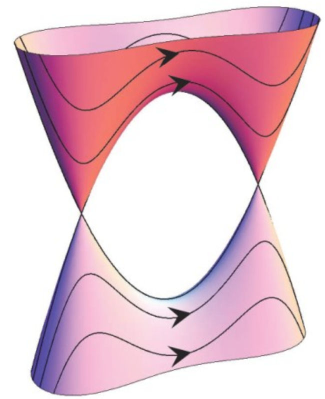


t=0.5 | t=1 | t'=1.414 | t'=2

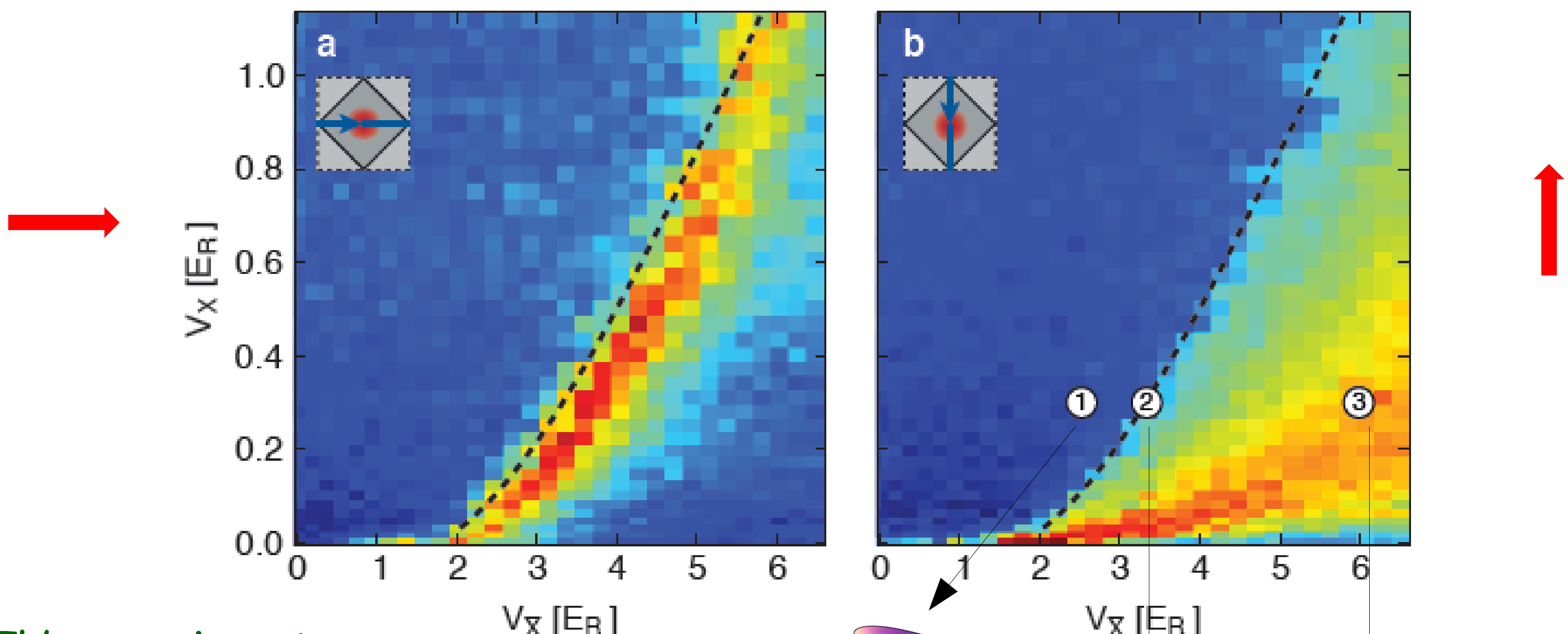
Measured transfered fraction of atoms: directions of motion

Single Dirac cone

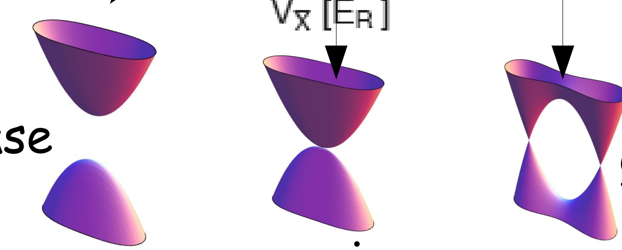




Double Dirac cone



ETH experiment



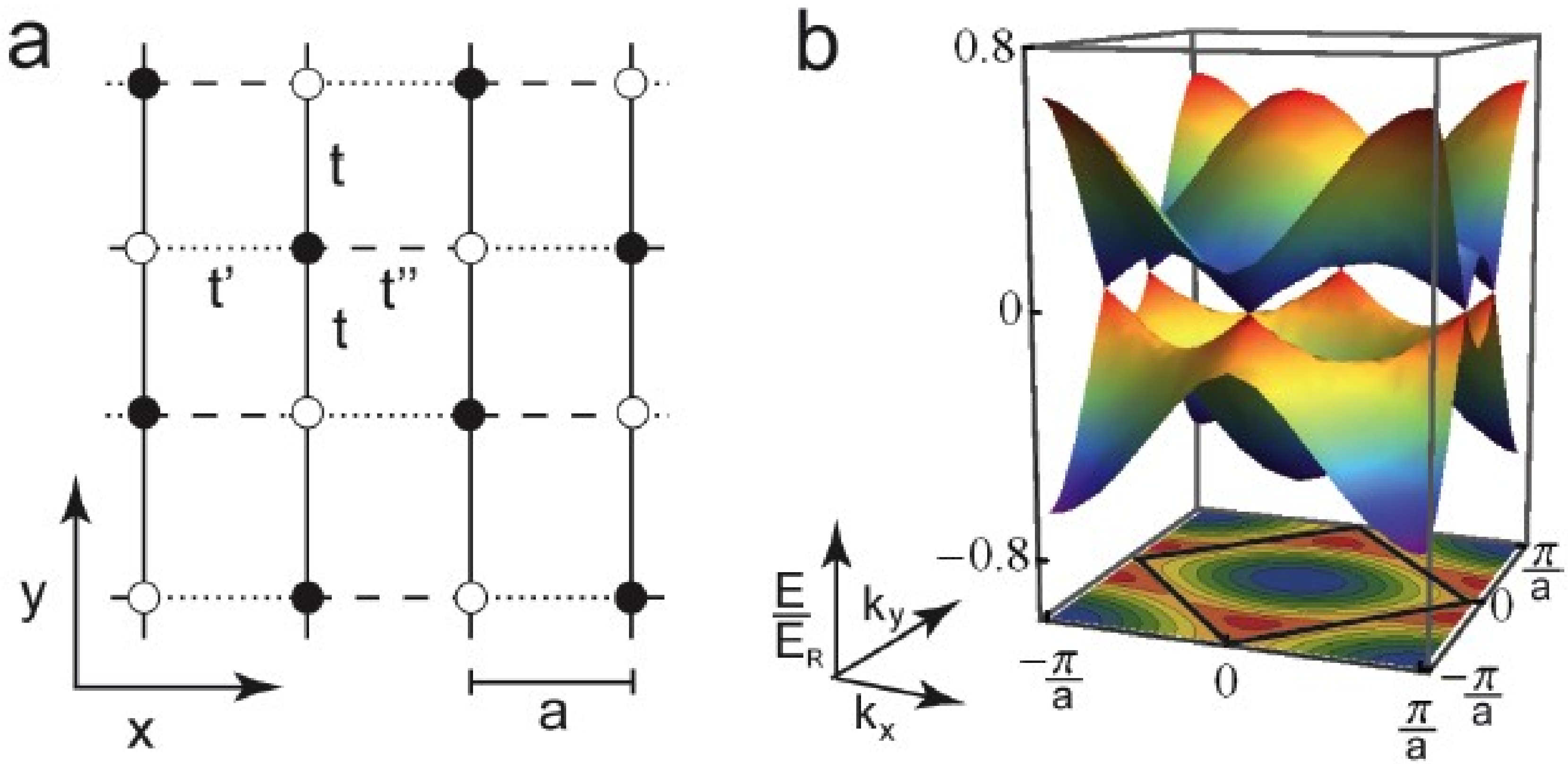
gapped G phase

merging

gapless D phase

5. Bloch-Zener oscillations across a merging transition of Dirac points: theory

L.K. Lim, J.N. Fuchs, G. Montambaux, to appear PRL 2012

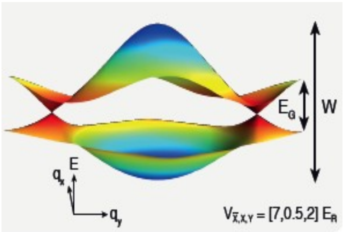


Anisotropic square lattice t-t'-t'' (or « leaking » brick-wall lattice): Dirac points, merging, gapped phase, 1D chains, lines of Dirac points (square lattice)

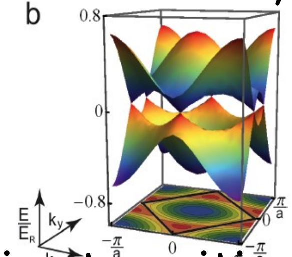
Numerical/Tight-binding/Effective band structures

Three levels of description for the band structure:

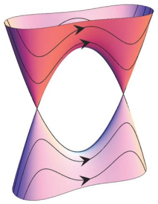
1) Ab-initio band structure from optical lattice potential: V_X, V_{Xb}, V_Y (laser intensities). Many bands.



2) Tight-binding model on an anisotropic square lattice: t,t',t'' (hopping amplitudes). Only two bands.

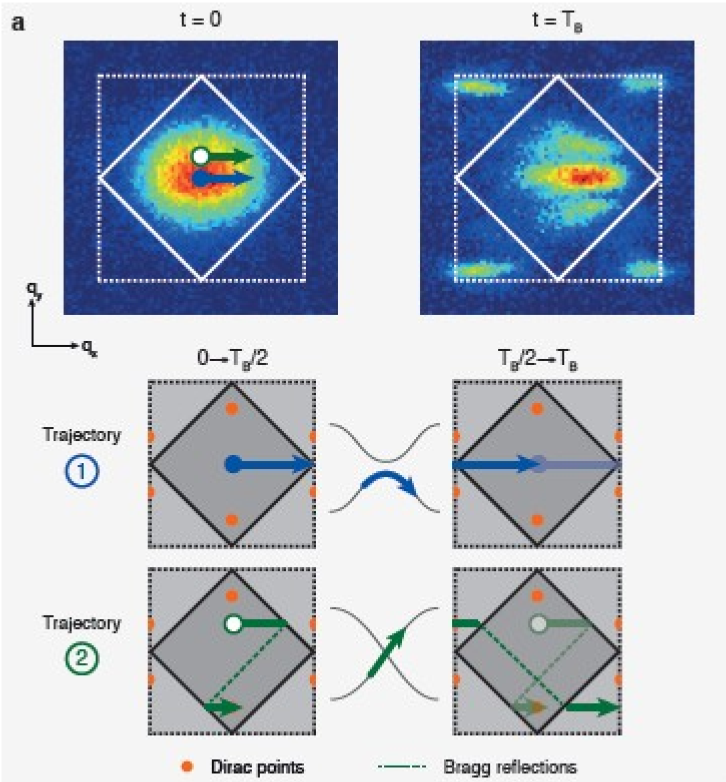


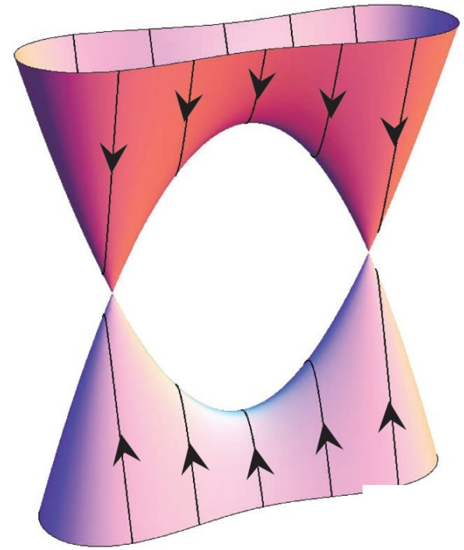
3) Universal hamiltonian describing the merging transition: Δ_*, c_x, m* (effective low energy parameters: merging gap, Dirac cone velocity, effective mass). Only two bands and only valid at low energy (far from the band edges).

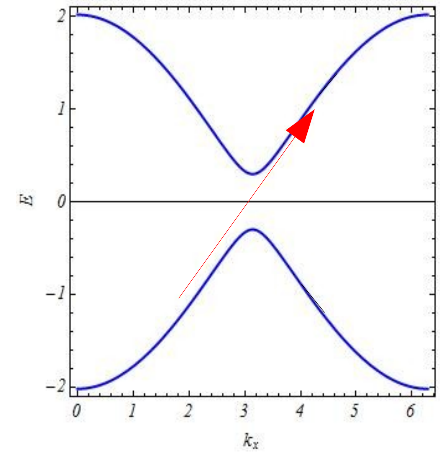


We compute the inter-band tunneling probability within this last description. First for a single atom and then for a trapped Fermi sea of non-interacting atoms.

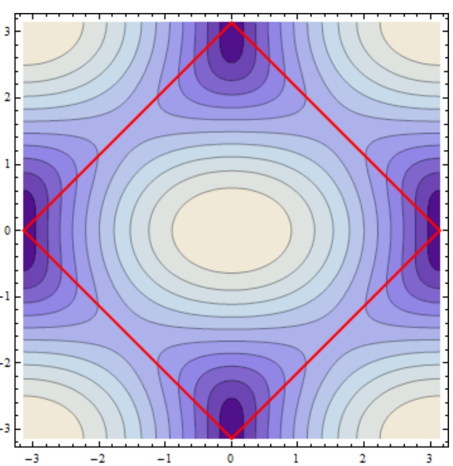
Single Dirac cone: single atom tunneling



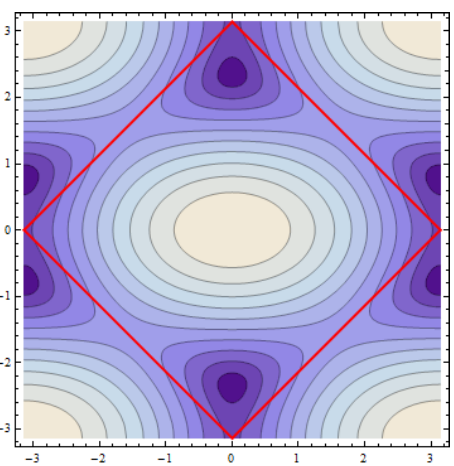




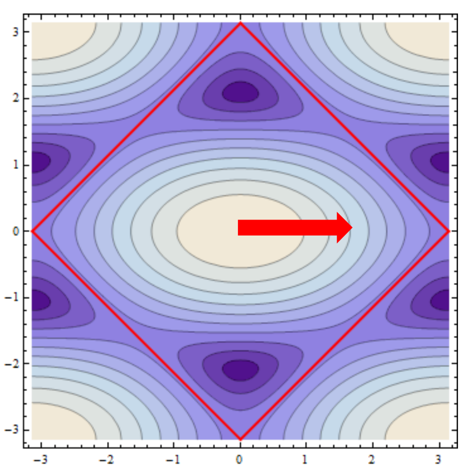
$$P_Z^x = e^{-\pi \frac{\left(\frac{q_y^2}{2m^*} + \Delta_*\right)^2}{c_x F}}$$



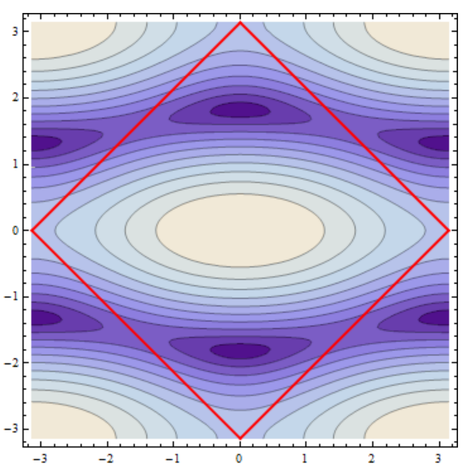
t'=2



t'=1.414

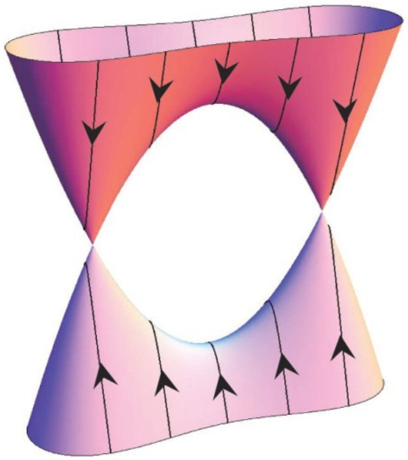


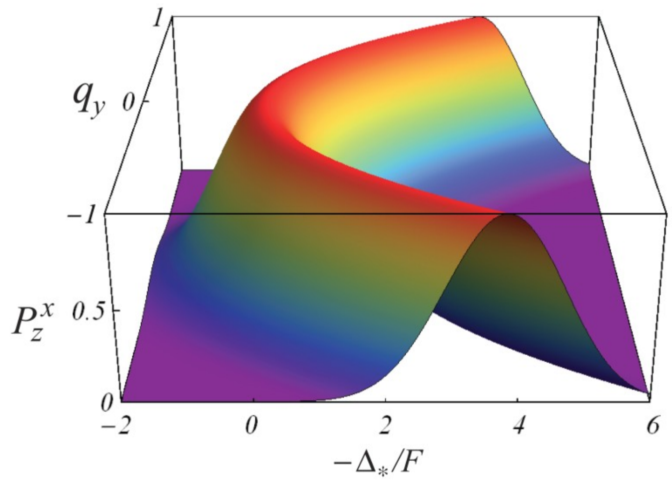
t=1



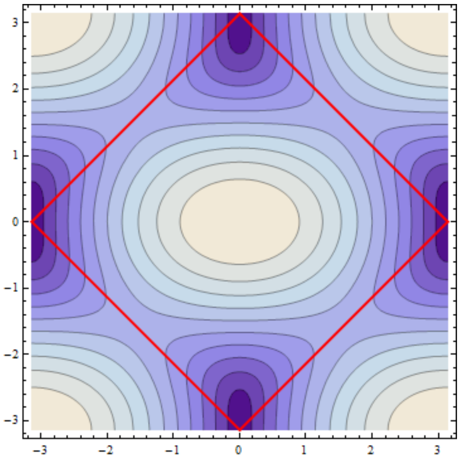
t=0.5

Single Dirac cone: single atom tunneling

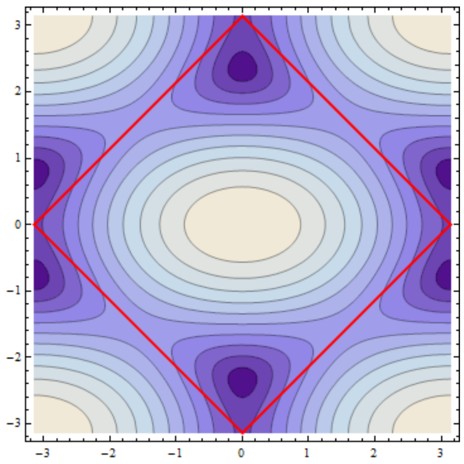




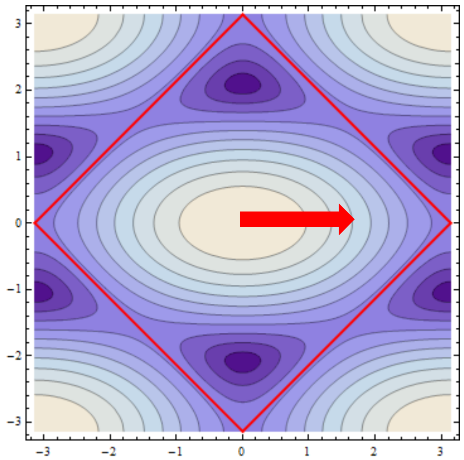
Transfer probability
as a function of q_y and Δ*



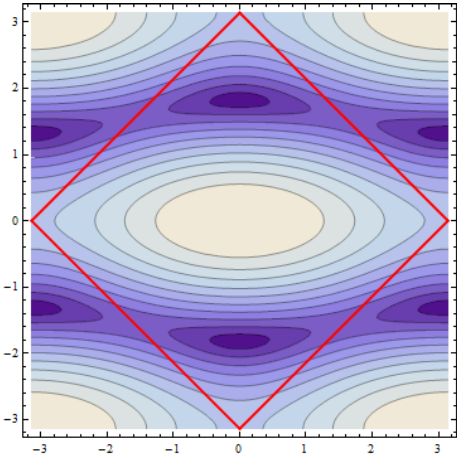
t'=2



t'=1.414

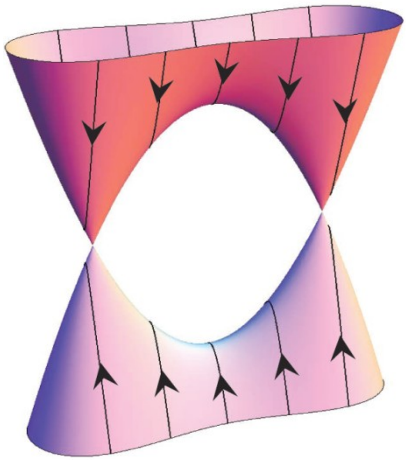


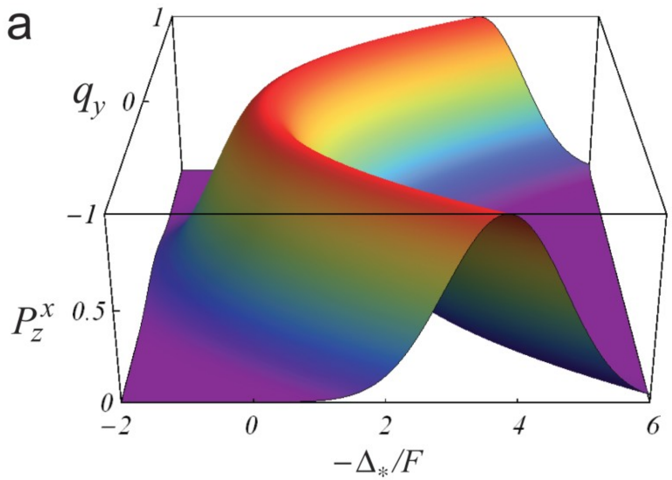
t=1



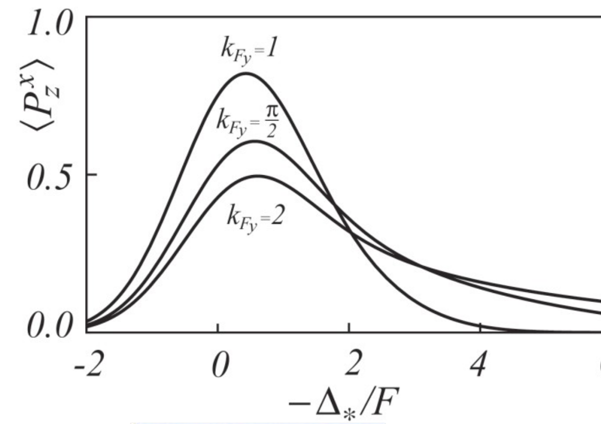
t=0.5

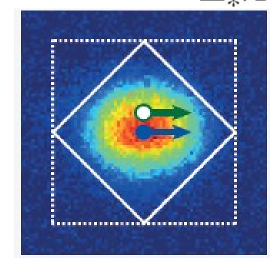
Single Dirac cone: Fermi sea tunneling



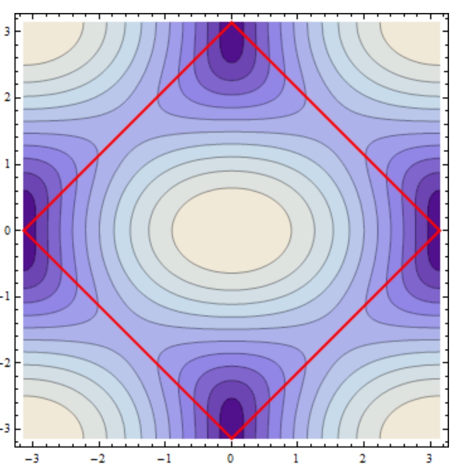


Transfer probability for a cloud of size k_{Fy}

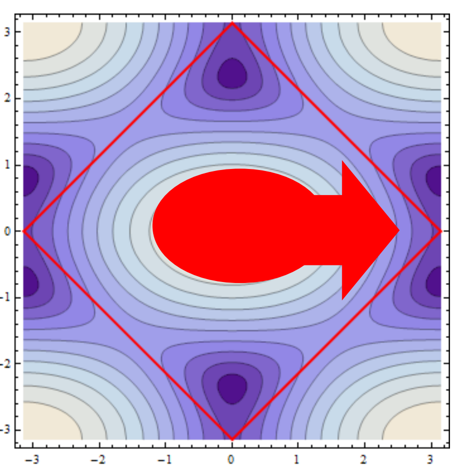




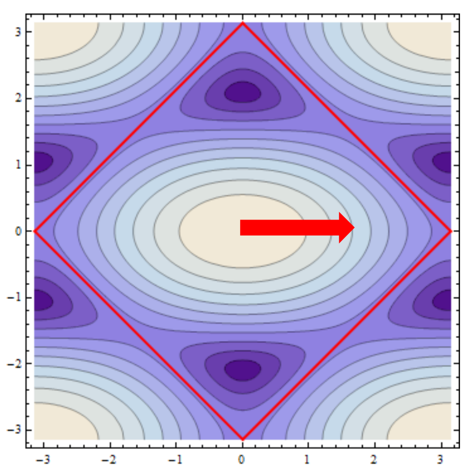
Transfer probability as a function of q_y and Δ*



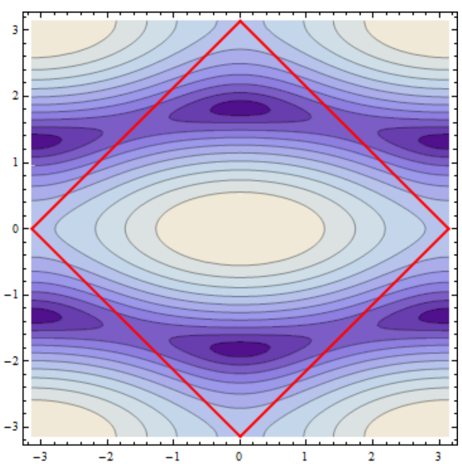
t'=2



t'=1.414

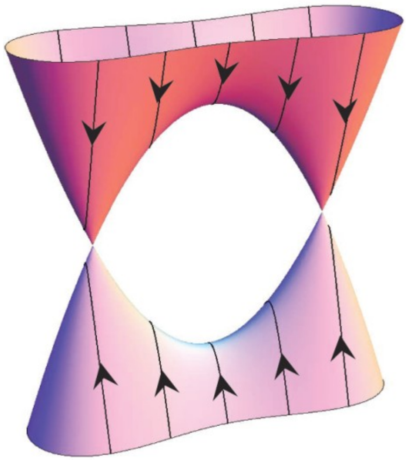


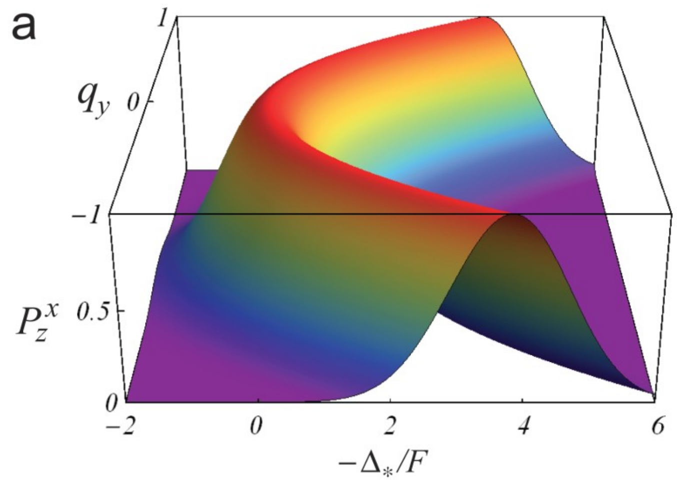
t=1



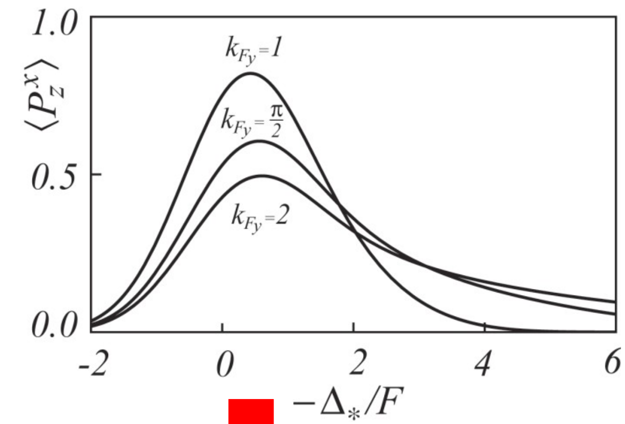
t=0.5

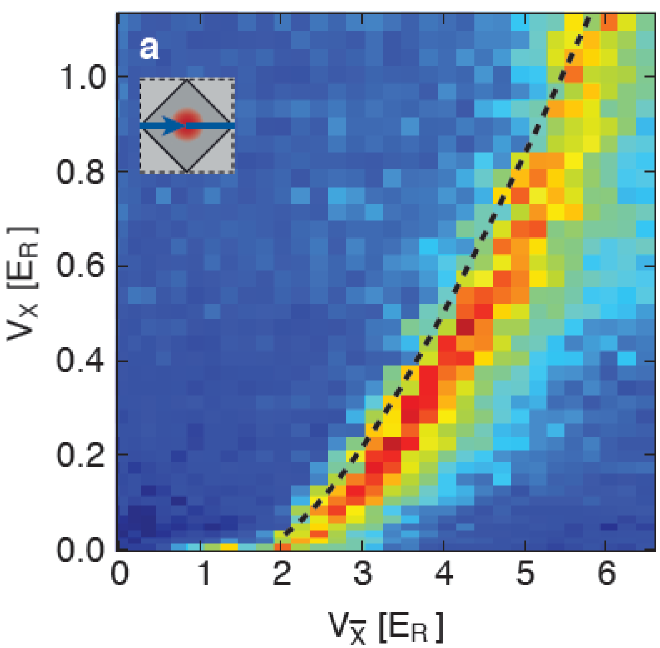
Single Dirac cone: Fermi sea tunneling





Transfer probability for a cloud of size k_{Fy}

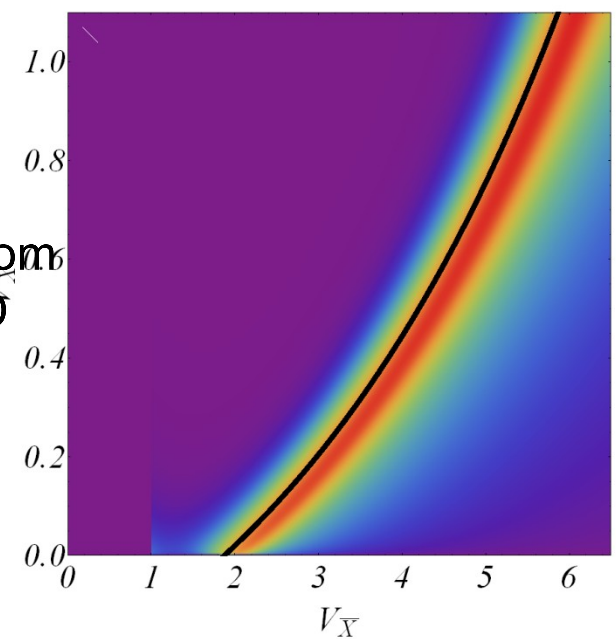




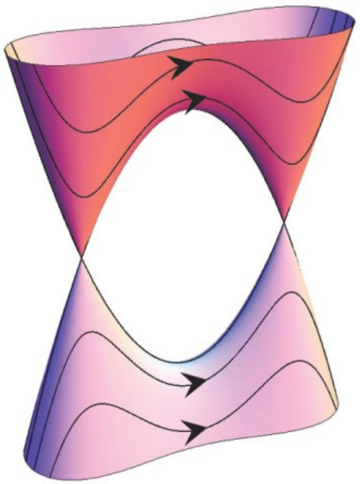
Line of maximum:

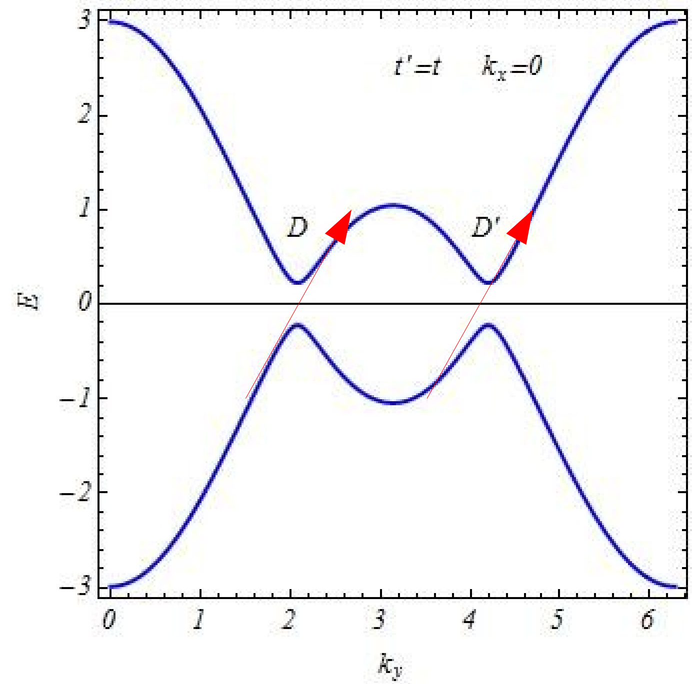
$P_z=1$

Merging transition from the G phase to the D phase



Double Dirac cone: single atom tunneling (incoherent)



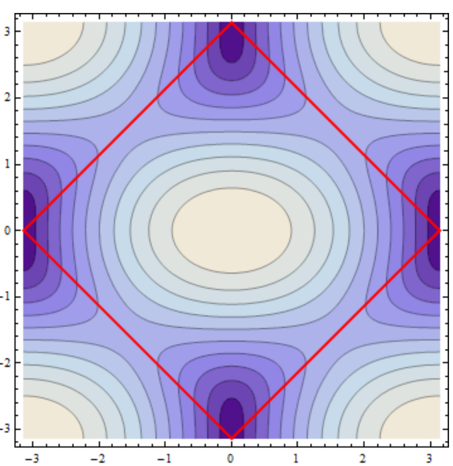


Combining probabilities gives:

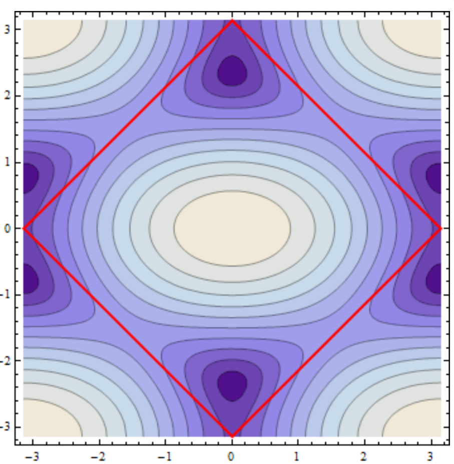
$$P_t^y = 2P_Z^y(1 - P_Z^y)$$

Non-monotonous function of P_Z. Maximum for P_z=1/2

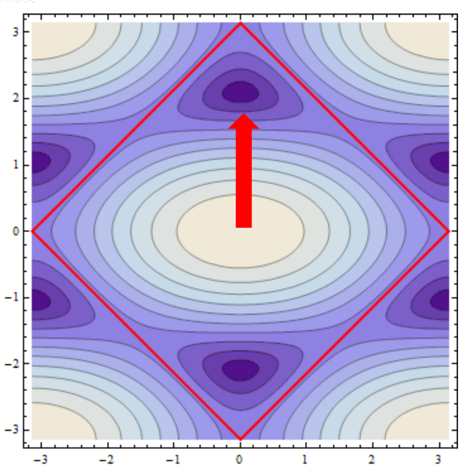
$$P_Z^y = e^{-\pi \frac{c_x^2 q_x^2}{c_y F}}$$



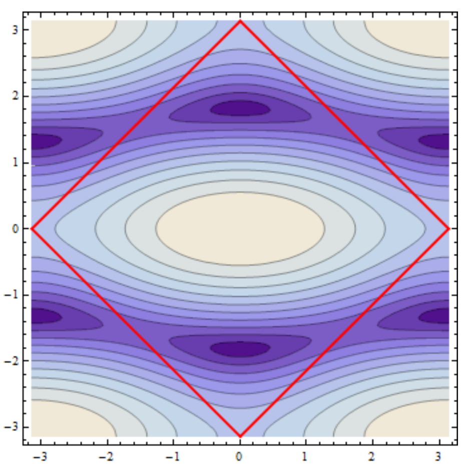
t'=2



t'=1.414

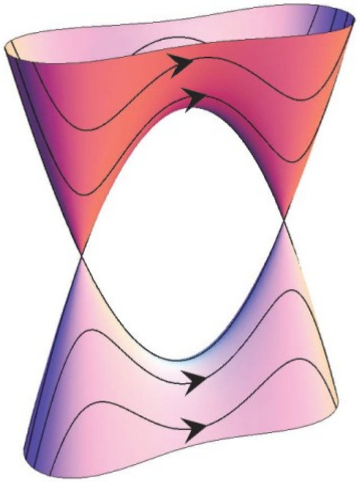


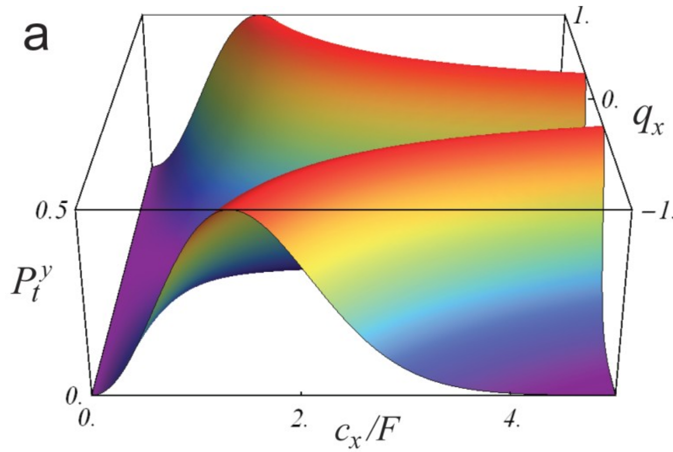
t=1



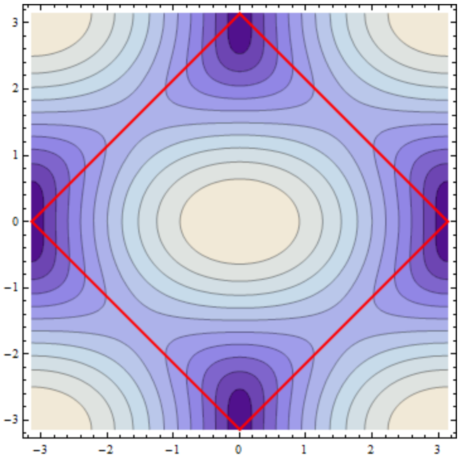
t=0.5

Double Dirac cone: single atom tunneling (incoherent)

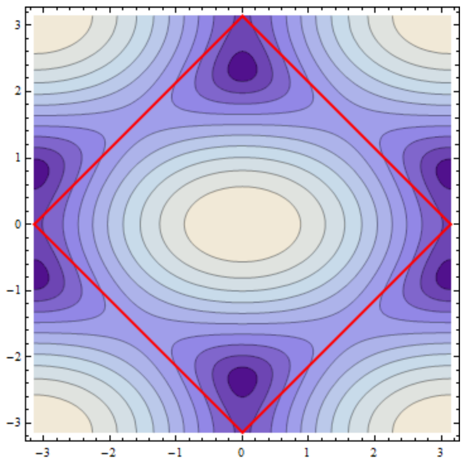




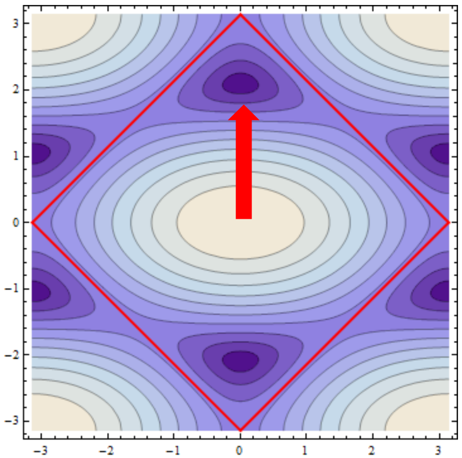
Transfer probability as a function of q_x, c_x



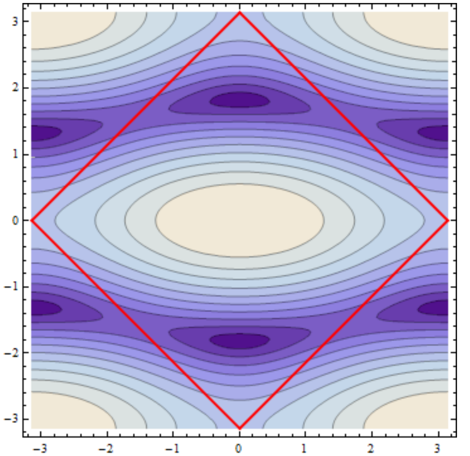
t'=2



t'=1.414

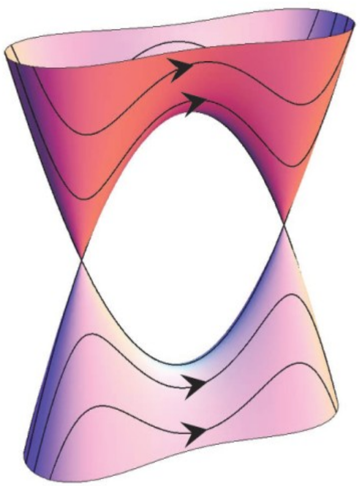


t=1

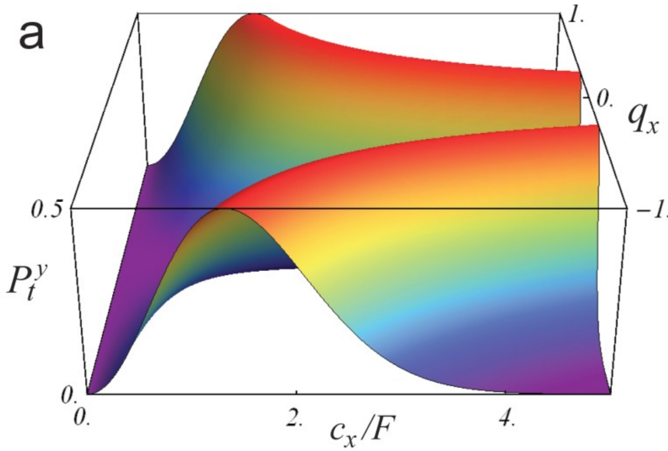


t=0.5

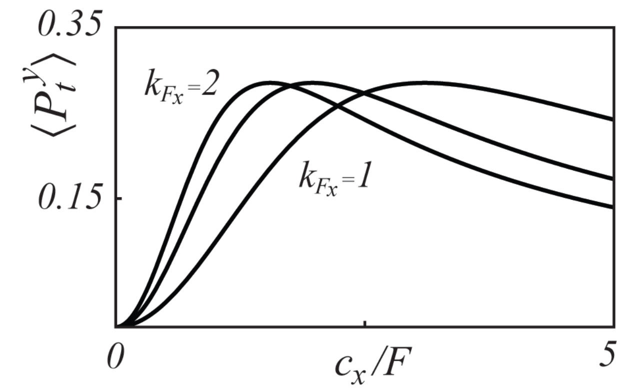
Double Dirac cone: Fermi sea tunneling (incoherent)

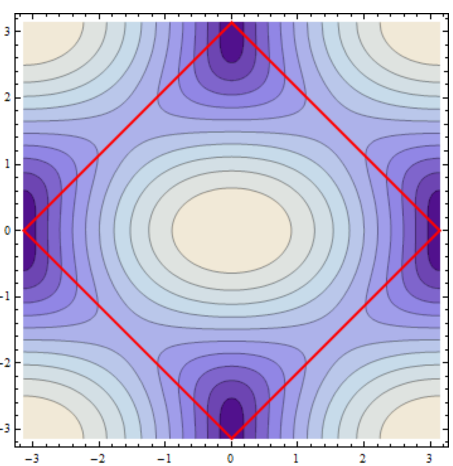


Transfer probability as a function of c_x

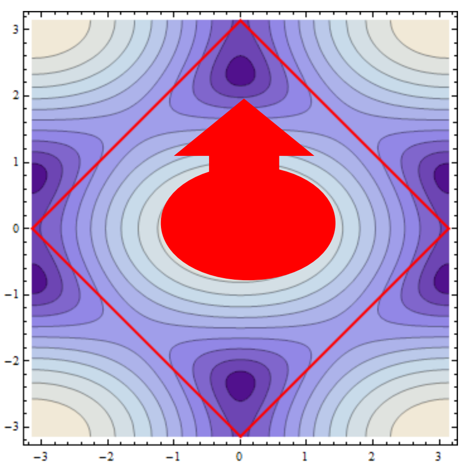


Transfer probability for a cloud of size k_{Fy}

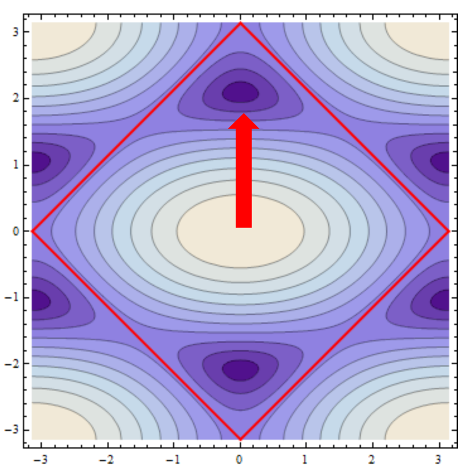




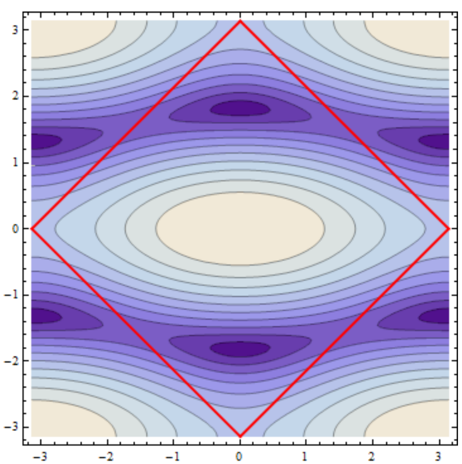
t'=2



t'=1.414



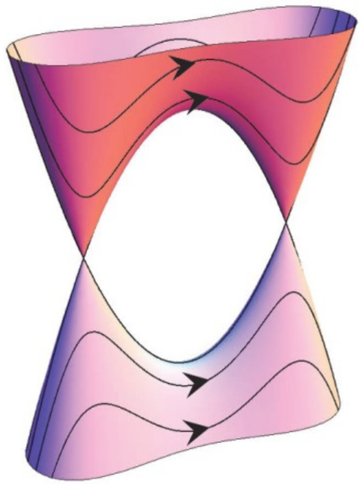
t=1

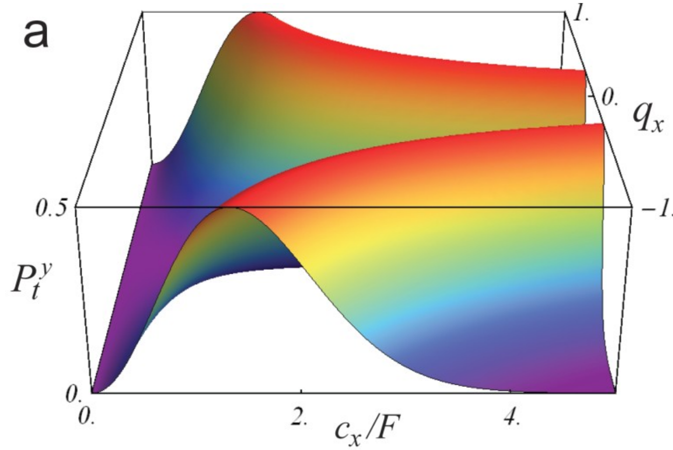


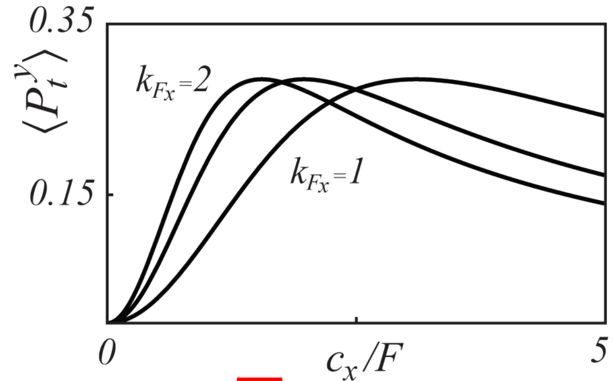
t=0.5

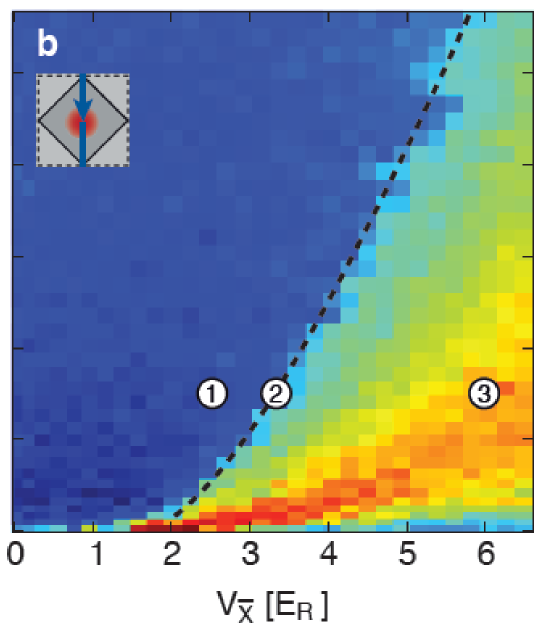
Double Dirac cone:
Fermi sea tunneling (incoherent)

Transfer probability for a cloud of size k_{Fy}





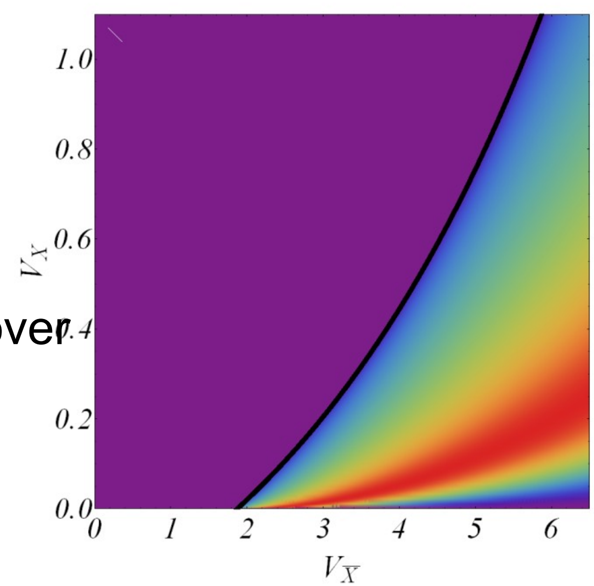




Line of maximum:

$P_z=1/2$

F-dependent crossover from the D phase to the L phase



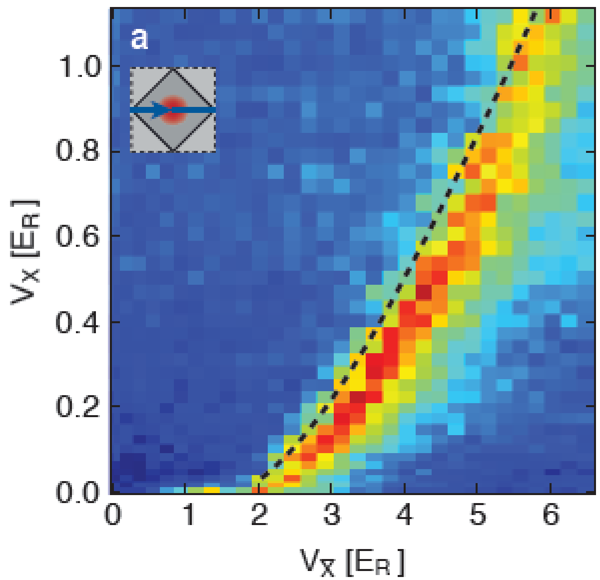
Summary

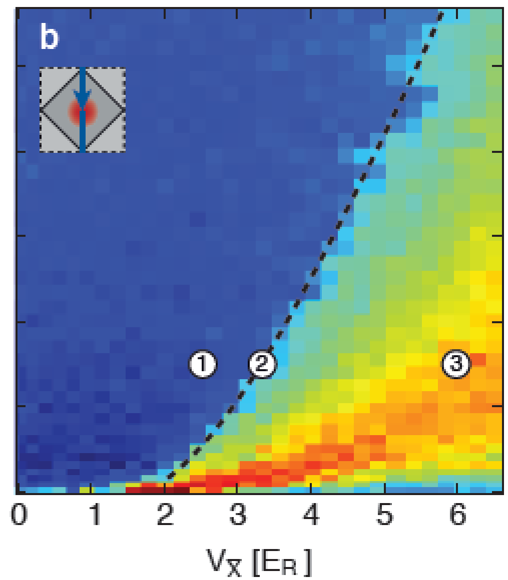
Single Dirac cone

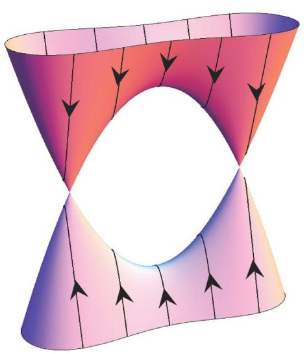
Double Dirac cone

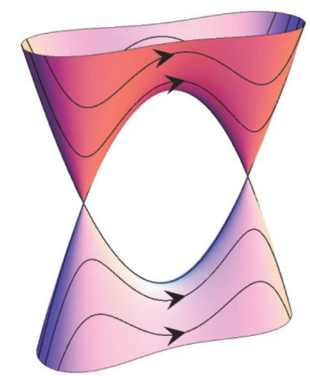
Experiment

Tarruel, Greif, Uehlinger, Jotzu, Esslinger Nature 2012



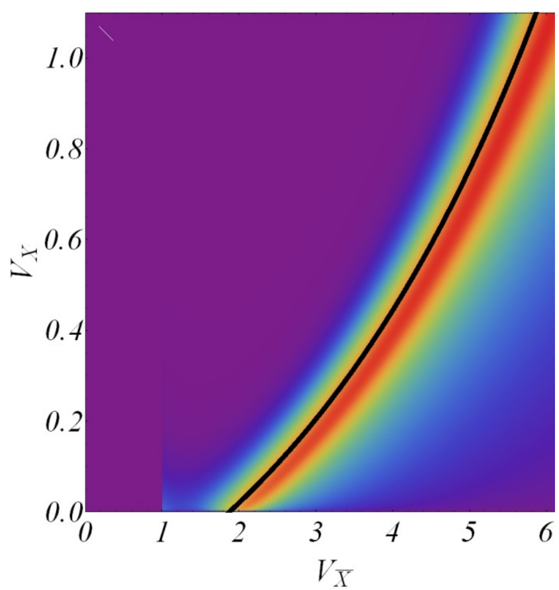


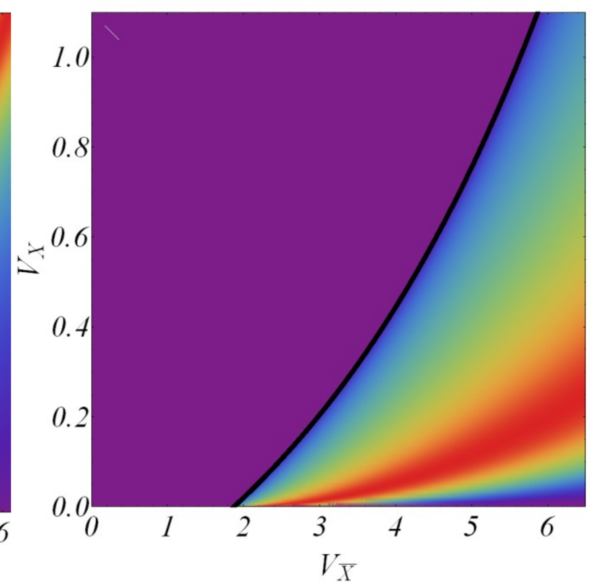




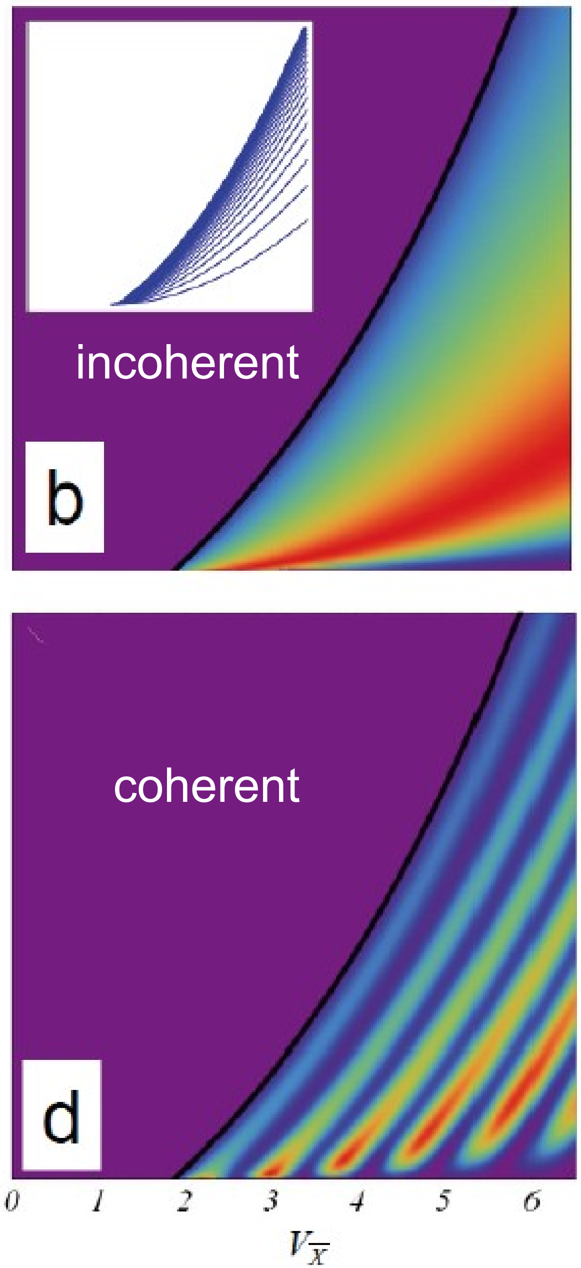
Theory

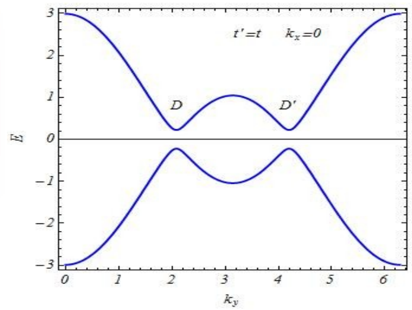
Lim, Fuchs, Montambaux, PRL 2012





Prediction: coherent double Dirac cone -> Stückelberg interferences





2 paths from lower to upper band:
#1 jump – stay
#2 stay - jump

Combining probability amplitudes gives:

$$P_t^y = 4P_Z^y(1 - P_Z^y)\cos^2(\varphi/2 + \varphi_d)$$

With the dynamical phase

$$\varphi = 2\int_0^{q_D} dq_y \sqrt{(\Delta_* + q_y^2/2m^*)^2 + c_x^2 q_x^2}/F$$

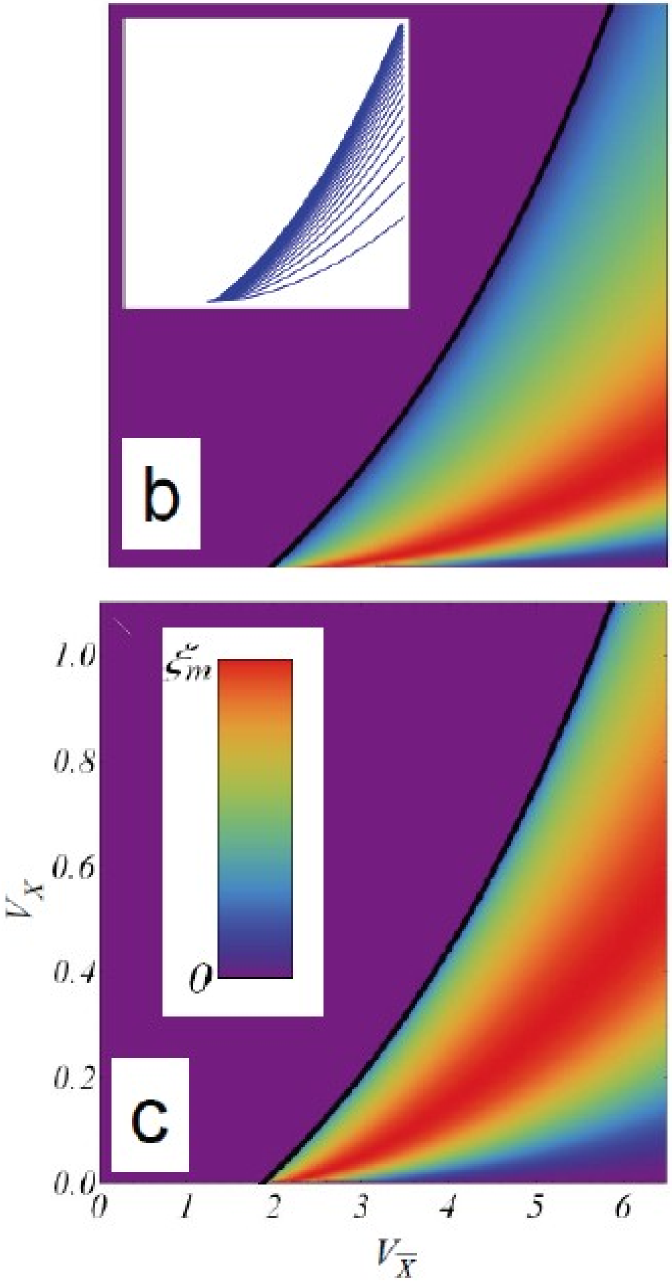
And the phase delay

$$\varphi_d = -\pi/4 + \delta(\ln\delta - 1) + \arg\Gamma(1 - i\delta)$$

See review Shevchenko, Ashab, Nori, Phys. Rep. 2010

In the ETH experiment, interferences are washed out by integration over the third spatial direction (z, along the tubes, detection process).
Interferences observable in a strictly 2D gas.

Prediction: Bloch-Zener oscillations with different forces *F*

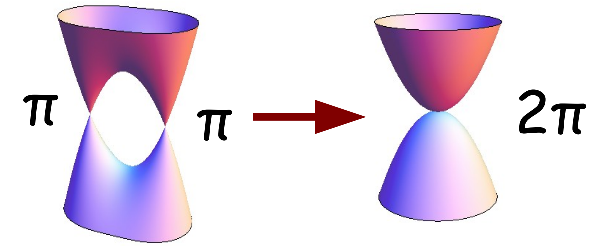


F as in the experiment

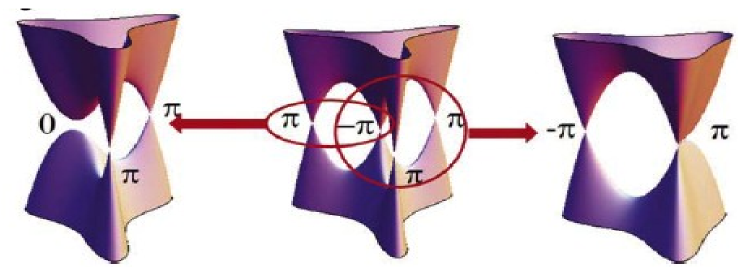
and 5 times larger

The « second red line » marking the crossover from the gapless Dirac phase to the gapless square phase (with lines of Dirac points) depends on the force.

6. Perspectives



Detection of other mergings (e.g. bilayer)
Splitting 2π-> (π,π) (Pomeranchuk instability, nematic state)
Multimerging (π,π,π,-π)->.....



See de Gail, Goerbig, Guinea, Montambaux, Castro-Neto, PRB 2011;
de Gail, Fuchs, Goerbig, Piéchon, Montambaux Physica B 2012.

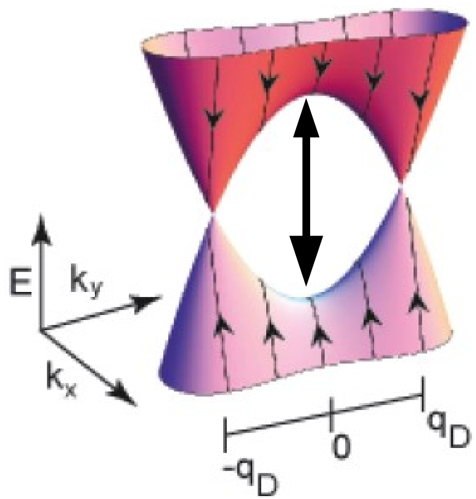
Several Bloch oscillations: more Stückelberg interferences, several frequencies in the Bloch-Zener oscillations
See e.g. Krueckl and Richter, PRB 2012

Landau-Zener tunneling with the complete universal hamiltonian

« Berrymeter »: mapping the Berry curvature in the whole Brillouin zone via the anomalous velocity contribution to the Bloch oscillations (adiabatic motion, no LZ tunneling).
See also Price and Cooper, PRA 2012

Thank you

Mapping tight-binding model on universal hamiltonian



Saddle point energy = $2|\Delta^*|$
Dirac cone velocities: c_x and c_y

$$\Delta_* = t' + t'' - 2t, \quad c_x = t' - t'', \quad c_y = \sqrt{4t^2 - (t' + t'')^2}$$

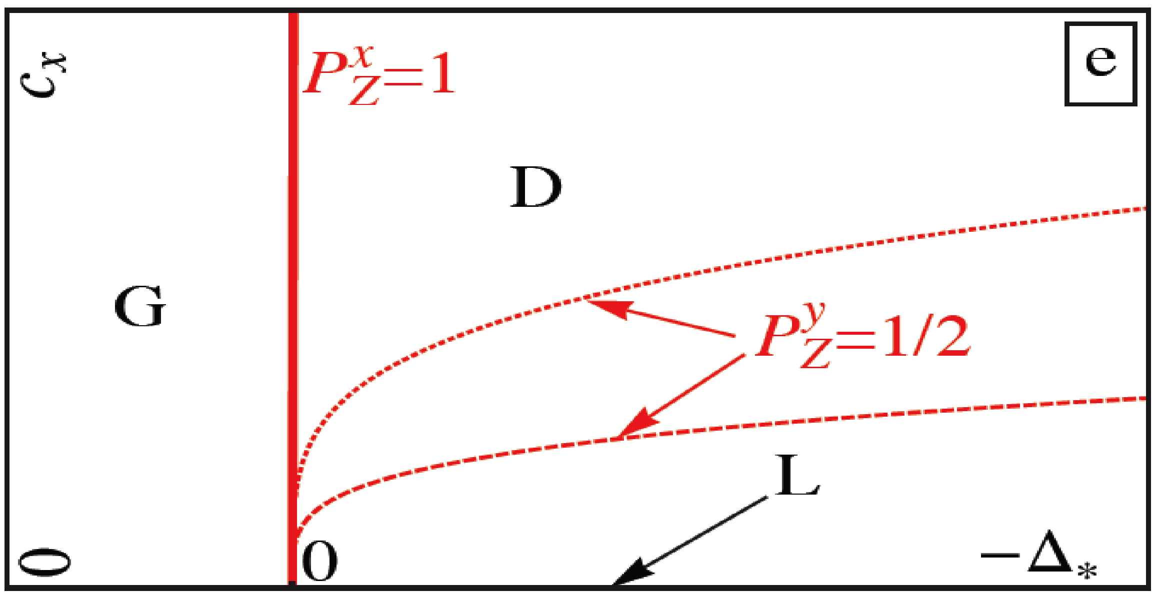
$$m^* = \frac{2}{2t + t' + t''}$$

$$q_D = \sqrt{-2m^* \Delta_*} = 2\sqrt{\frac{2t - t' - t''}{2t + t' + t''}}$$

Phase diagram

Anisotropic square lattice t-t'-t'': Dirac points, merging, gapped phase, 1D chains, lines of Dirac points (square lattice)

t'-t''



2t-(t'+t'')
driving parameter for the merging